C.D.S.
Combined Defence Services

EXAM I 2024

Elementary Mathematics

Time Allowed: 2 Hours **Maximum Marks: 100**

Instructions

1. *This Test Booklet contains **100** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.*
2. *You have to mark all your responses **ONLY** on the separate Answer Sheet provided. See directions in the Answer Sheet.*
3. ***All** items carry equal marks.*
4. ***Penalty for wrong answers:***
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   (i) *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.*
   (ii) *If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   (iii) *If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.*

1. If $a : b : c : d = \sqrt{4} : \sqrt{3} : \sqrt{2} : \sqrt{1}$, then what is the value of $\frac{(-a^2+b^2+c^2+d^2)}{(a^2-b^2+c^2-d^2)}$?

(a) 1 **(b)** 2 **(c)** 3 **(d)** 6

2. The speeds of four cars are $2u$, $3u$, $4u$ and xu and the time taken by them to cover the same distance is xt, $4t$, $3t$ and $2t$ respectively, where x, u, t are real numbers. What is the value of x?

(a) 8 **(b)** 6 **(c)** 5 **(d)** 2

3. If $m : n = 1 : 2$ and $p : q = 3 : 4$, then what is $(2m + 4p) : (n + 3q)$ equal to?

(a) 1 : 1 **(b)** 1 : 3
(c) 2 : 1 **(d)** 2 : 3

4. If the rate of interest is 5%, then what would be the difference between compound interest and simple interest received on ₹ 10,000 (each) after 3 years from now?

(a) ₹ 175.25 **(b)** ₹ 152.25
(c) ₹ 76.25 **(d)** ₹ 24.25

5. A person bought a book at $\frac{3}{4}^{\text{th}}$ of its listed price and sold it at 50% more than its listed price. What is the percentage of gain in the transaction?

(a) 20% **(b)** 40% **(c)** 75% **(d)** 100%

6. If the difference between the interior and exterior angles of a regular polygon is 144°, then what is the number of sides of the polygon ?

(a) 12 **(b)** 16 **(c)** 18 **(d)** 20

7. If the sum and product of the roots of a quadratic equations are 2 and – 100 respectively, then which one of the following is correct?

(a) There are infinitely many such equations having different roots.
(b) There is only one such equation which is $x^2 + 2x - 100 = 0$.
(c) There is only one such equation which is $x^2 - 2x - 100 = 0$.
(d) There is no such equation.

8. If 2 is a zero of the polynomial $p(x) = x^3 + 3x^2 - 6x - a$, then what is the sum of the squares of the other zeros of the polynomial ?

(a) 10 **(b)** 17 **(c)** 21 **(d)** 37

9. If $t = \cos 79°$, then what is $\text{cosec } 79° (1 - \cos 79°)$ equal to ?

(a) $\sqrt{\frac{1+t}{1-t}}$ **(b)** $\frac{t}{\sqrt{1-t^2}}$

(c) $\frac{\sqrt{1-t^2}}{t}$ **(d)** $\sqrt{\frac{1-t}{1+t}}$

10. Suppose $p(x) = x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$ and $q(x) = x^4 + b_3x^3 + b_2x^2 + b_1x + b_0$ are the polynomials. If α, β, γ, δ are zeros of $p(x)$ and α, β, γ, λ are zeros of $q(x)$, then what is $\frac{p(x)-q(x)}{(x-\alpha)(x-\beta)(x-\gamma)}$ equal to ?

(a) $-\lambda + \delta$ **(b)** $\lambda - \delta$
(c) $\lambda + \delta$ **(d)** $-\lambda - \delta$

11. If the equation $x\cos\theta = x^2 + p$ has a real solution for every θ where $0 \le \theta \le \frac{\pi}{4}$, then which one of the following is correct?

(a) $p = \frac{1}{8}$ **(b)** $p \le \frac{1}{8}$
(c) $p \ge \frac{1}{8}$ **(d)** $p \le \frac{1}{4}$

12. What is the difference between the greatest value and the least value of $\cos^2\theta + 3\sin^2\theta + 2$?
(a) 4 **(b)** 3 **(c)** 2 **(d)** 1

13. ABC is a right-angled triangle, right-angled at B such that AB = 6 cm and BC = 8 cm. What is the perimeter of the square inscribed in the triangle ABC with maximum area?
(a) $\frac{24}{7}$ cm **(b)** $\frac{96}{7}$ cm
(c) 24cm **(d)** 32 cm

14. What is the greatest value of k for which $2x^2 - 4x + k = 0$ has real roots ?
(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

15. Consider the following data : 110, 41, 43, 95, 127, 99, 61, 92, 71, 93, 110, 36. If 98 is replaced by 94, then consider the following statements:
1. The difference between new median and old median is 1.
2. The difference between new mean and old mean is less than 0.1.
3. The difference between new mode and old mode is zero.
Which of the statements given above are correct?
(a) l and 2 only **(b)** 2 and 3 only
(c) l and 3 only **(d)** 1, 2 and 3

16. What is the digit at the 100^{th} place of number $(225)^{40}$?
(a) 6 **(b)** 5 **(c)** 4 **(d)** 2

17. If a, b, c, d are natural numbers, then how many possible remainders are there when $1^a + 2^b + 3^c + 4^d$ is divided by 10?
(a) 3 **(b)** 4 **(c)** 5 **(d)** 6

18. If n is a natural number, then what is the sum of all distinct remainders of $4^n + 6^n + 9^n + 11^n$ when divided by 10 for various values of n ?
(a) 3 **(b)** 4 **(c)** 6 **(d)** 7

19. When the number (12345678910111213 ... 99100) is divided by 16, what will be the remainder ?
(a) 15 **(b)** 12 **(c)** 4 **(d)** 3

20. A, B, C, D can complete a work in 3, 6, 9, 12 hours respectively. Further, only one person can work at a time in each hour and nobody can work for two consecutive hours. It is not necessary to engage all. What is the minimum number of hours that they will take to finish the work?
(a) $\frac{36}{25}$ **(b)** $\frac{12}{5}$
(c) 4 **(d)** 2

21. If $p = \sqrt[3]{\left(a+\sqrt{a^2+b^3}\right)} + \sqrt[3]{\left(a-\sqrt{a^2+b^3}\right)}$, then what is $p^3 +$
(a) $-2a$ **(b)** a **(c)** $2a$ **(d)** $3a$

22. A plank of wood 4.25 m long and 3.4 wide is to be cut into square pieces of equal size. How many square pieces of largest size can be cut from the plank, if no wastage is allowed?
(a) 45 **(b)** 20 **(c)** 400 **(d)** 500

23. What is the HCF of $x^4 - 13x^2y^2 - 300y^4$, $x^3 - 4x^2y - 4xy^2 - 5y^3$ and $x^3 - 125y^3$?
(a) $x - 5y$ **(b)** $x + 5y$
(c) $x^2 + 5xy + 25y^2$ **(d)** 1

24. If HCF of 768 and x^6y^2 is $32xy$ for natural numbers $x \ge 2$, $y \ge 2$, then what is the value of $(x + y)$?
(a) 5 **(b)** 7 **(c)** 9 **(d)** 11

25. What is the smallest natural number n such that $(n + 1) \times n \times (n - 1) \times (n - 2) \times \ldots 3 \times 2 \times 1$ is divisible by 910?
(a) 91 **(b)** 90 **(c)** 13 **(d)** 12

26. The expression $555^{777} + 777^{555}$ is divisible by which of the following?
1. 2 **2.** 3 3. 37
Select the correct answer using the code given below :
(a) l and 2 only **(b)** 2 and 3 only
(c) l and 3 only (d) 1, 2 and 3

27. Consider the following statements:
1. If $(3m^3 + 2m^2 + 5m + n)/m$ is not an integer, where m and n are integers, then n is not divisible by m.
2. $5(8^m) + 2^{3m}$ is divisible by 48 for all whole numbers m.
Which of the statements given above is/are correct ?
(a) l only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

28. The sum of two positive numbers is 40. If the GM of these two numbers is lower than their AM by 20%, then what is the difference between the two numbers?
(a) 12 **(b)** 18 (c) 24 **(d)** 28

29. 50 men can complete a work in 40 days. They begin the work together but a batch of 5 men left after each period of 10 days. What is the time to complete the work?
(a) 45 days **(b)** 50 days
(c) 55 days **(d)** 60 days

30. If $x = \cfrac{1}{2+\cfrac{3}{4+\cfrac{5}{6+\cfrac{7}{8+\cfrac{9}{10}}}}}$
then which one of the following is correct ?
(a) $0 < x < 0.5$ **(b)** $x = 0.5$
(c) $0.5 < x < 1.0$ **(d)** $x > 1.0$

31. A bottle contains spirit and water in the ratio 1: 4 and another identical bottle contains spirit and water in the ratio 4 : 1. In what ratio should the mixtures in the two bottles be mixed to get a new mixture in which the ratio of spirit to water is 1 : 3?
(a) 5 : 1 **(b)** 6 : 1
(c) 10 : 1 **(d)** 11 : 1

32. If $3\sin\theta + 5\cos\theta = 5$, then what is the value of $5\sin\theta - 3\cos\theta$?
(a) –3 **(b)** –2 **(c)** 5 **(d)** 8

33. Consider the following in respect of the polynomial $x^{4k} + x^{4k+2} + x^{4k+4} + x^{4k+6}$:
1. The remainder is zero when the polynomial is divided by $x^2 + 1$.
2. The remainder is zero when the polynomial is divided by $x^4 + 1$.
Which of the statements given above is/are correct ?
(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

34. What is the minimum value of
$\dfrac{\sin^2 A + 5\sin A + 1}{\sin A}$ **for** $0 < A \le \dfrac{\pi}{2}$?
(a) 3 **(b)** 5
(c) 7 **(d)** 9

35. What is $\dfrac{3}{1^2\times 2^2} + \dfrac{5}{2^2\times 3^2} + \dfrac{7}{3^2\times 4^2} \ldots$ equal to?
(a) 1 **(b)** 4 **(c)** 7 **(d)** 9

36. If $\cfrac{1}{a+\cfrac{1}{b+\cfrac{1}{c+\cfrac{1}{d+\cfrac{1}{e}}}}} = \dfrac{421}{972}$,
then what is the value of $a \times b \times c \times d \times e$?
(a) 720 **(b)** 480 **(c)** 360 **(d)** 60

37. A cube whose edge is 14 cm long has on each of its faces a circle of 7 cm radius painted yellow. What is the total area of unpainted surface?
(Take $\pi = \dfrac{22}{7}$)
(a) 126 square cm **(b)** 189 square cm
(c) 252 square cm **(d)** 315 square cm

38. From a circular metal plate of radius 7 cm and thickness 0.16 mm, a sector is cut off containing an angle 150°. The remaining piece is moulded into a spherical bead of radius r. What is the value of r in cm?
(a) 0.35 **(b)** 0.7 **(c)** 1.05 **(d)** 1.4

39. The chord AB of a circle with centre at O is $2\sqrt{3}$ times the height of the minor segment. If P is the area of the sector OAB and Q is the area of the minor segment of the circle, then what is the approximate value of $\dfrac{P}{Q}$?
(Take $\sqrt{3} = 1.7$ and $\pi = 3.14$)
(a) 1.4 **(b)** 1.7 **(c)** 2.2 **(d)** 2.6

40. What is the area of the region between two concentric circles, if the length of a chord of the outer circle touching the inner circle at a particular point of its circumference is 14 cm?
$\left(\text{Take } \pi = \dfrac{22}{7}\right)$
(a) 154 square cm
(b) 144 square cm
(c) 132 square cm
(d) Cannot be determined due to insufficient data

41. In a right-angled triangle ABC, AB = 15 cm, BC = 20 cm and AC = 25 cm. Further, BP is the perpendicular on AC. What is the difference in the area of triangles PAB and PCB?
(a) 40 square cm **(b)** 42 square cm
(c) 45 square cm **(d)** 48 square cm

42. Let the positive numbers $a_1, a_2, a_3, \ldots, a_{3n}$ be in GP. If P is the GM of $a_1, a_2, a_3, \ldots . a_n$, and Q is the GM of $a_{n+1}, a_{n+2}, a_{n+3}. \ldots, a_{3n}$, then what is the GM of $3n$ numbers ?
(a) P^2Q **(b)** PQ^2
(c) $\sqrt{PQ}$ **(d)** $P^{\frac{1}{3}}Q^{\frac{2}{3}}$

43. The cost price of y articles is equal to selling price of z articles. If $y : z = 5 : 4$, what is the profit percentage?
(a) 20% **(b)** 25% **(c)** 30% **(d)** 40%

44. A sum of money invested at simple interest

triples itself in 8 years and becomes n times in 20 years. What is the value of n?
(a) 5 **(b)** 6 **(c)** 7.5 **(d)** 9

45. If the work done by x men in $(x + 1)$ days is equal to the work done by $(x + 5)$ men in $(x - 2)$ days, then what is the value of x ?
(a) 5 **(b)** 6 **(c)** 7 **(d)** 8

46. If $(a + b) : (b + c) : (c + a) = 5 : 7 : 6$, then what is the value of $(a - b + c) : (a + b - c)$?
(a) 1 : 1 **(b)** 2 : 3 **(c)** 3 : 1 **(d)** 4 : 3

47. Let x be the compound interest at the end of 3 years on a sum of ₹ 1000 at the rate of 10% compounded annually and y be the simple interest at the end of 3 years on a sum of ₹ 1000 at the annual rate of 11%. What is the difference between x and y ?
(a) ₹ 16 **(b)** ₹ 15 **(c)** ₹ 5 **(d)** ₹ 1

48. In a quadrilateral ABCD, AB = 6 cm, BC = 18 cm, CD = 6 cm and DA = 10 cm. If the diagonal BD = x, then which one of the following is correct ?
(a), $8 < x < 12$ **(b)** $12 < x < 16$
(c) $16 < x < 18$ **(d)** $18 < x < 20$

49. In a quarter circle of radius R, a circle of radius r is inscribed. What is the ratio of R to r?
(a) $(\sqrt{2}+1):1$ **(b)** $(\sqrt{3}+1):1$
(c) 3 : 2 **(d)** 5 : 4

50. In a quadrilateral ABCD, AB = BC and CD = DA; AC and BD are diagonals such that AC = 6 cm and BD = 12 cm. What is the area of the quadrilateral?
(a) 24 square cm **(b)** 30 square cm
(c) 36 square cm **(d)** 40 square cm

51. If $\tan(3A) = \cot(A - 22°)$, where 3A is an acute angle, then what is the value of A ?
(a) 25° **(b)** 27° **(c)** 28° **(d)** 30°

52. If $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = p\sec\theta + q\tan\theta$,

where $0 < \theta < \frac{\pi}{2}$, then what is $p + q$ equal to?
(a) 0 **(b)** 1 **(c)** 2 **(d)** 4

53. The angles of elevation of the top of a tower from two points A and B at a distance of x m and $(x + 5)$ m from the base of the tower of height 6 m and in the same straight line with it are complementary. What is the value of x ?
(a) 4 m **(b)** 5 m **(c)** 6 m **(d)** 9 m

54. Consider the following statements:
1. In a triangle ABC, if $\sin A + \sin B + \sin C = \frac{3\sqrt{3}}{2}$, then the triangle can be equilateral.
2. In a triangle ABC, if $\cos A + \cos B + \cos C = \frac{3}{2}$, then the triangle can be equilateral.
Which of the statements given above is/are correct?
(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

55. Two trains A and B leave Delhi for Hyderabad at 7:00 a.m. and 7:50 a.m. on the same day and travel at 80 kmph and 100 kmph respectively. After how many kilometers from Delhi will the two trains be together?
(a) $\frac{200}{3}$ km **(b)** 100 km
(c) $\frac{400}{3}$ km **(d)** $\frac{1000}{3}$ km

56. The length, breadth and height of a cuboid are increased by 10%, 20% and 50% respectively. What is the percentage increase in volume of the cuboid ?
(a) 100% **(b)** 99% **(c)** 98% **(d)** 50%

57. ₹ 9400 is distributed among P, Q, R in such a way that if ₹ 93, ₹ 24, ₹ 55 are deducted from their respective shares, then they have money in the ratio 3 : 4: 5. What is the share of P?
(a) ₹ 2307 **(b)** ₹ 2376
(c) ₹ 2508 **(d)** ₹ 2896

58. If P^2 varies as R and Q^2 varies as R, $(P \neq Q)$, then which of the following are correct ?
1. $P^2 + Q^2$ varies as R.
2. PQ varies as R.
3. $P^2 - Q^2$ varies as R.
Select the correct answer using the code given below :
(a) 1 and 2 only **(b)** 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

59. p number of men can finish a piece of work in q days. If there are 50% more men, then the work will be finished 12 days earlier. What is the value of q?
(a) 48
(b) 40
(c) 36
(d) Cannot be determined due to insufficient data

60. What is the minimum value of $\left(\frac{a^2+3a+1}{a}\right)\left(\frac{b^2+3b+1}{b}\right)$ for $a, b > 0$?
(a) 1 **(b)** 9 **(c)** 16 **(d)** 25

Consider the following for the next ten (10) items that follow :

Each item contains a question followed by two Statements. Answer each item using the following instructions :

Choose option

(a) If the Question can be answered by one of the Statements alone, but not by the other.

(b) If the Question can be answered by either Statement alone.

(c) If the Question can be answered by using both the Statements together, but cannot be answered by using either Statement alone.

(d) If the Question cannot be answered even by using both Statements together.

61. Let a, b, c and d be positive integers.
Question : Which one of a, b, c, d is closest to the product abcd ?
Statement-I: $a > b > c$
Statement-II: c is not the smallest.

62. Let $mn = k$, where m and n are prime numbers and k is an even number.
Question : What is the value. of $mn - n + 1$?
Statement-I: $m > n$
Statement-II: One of the numbers is 2.

63. Question: If p is a positive integer, then what is the remainder when p^n is divided by $p+1$?
Statement-I: n is even.
Statement-II: p is even.

64. Question : Is xy positive ?
Statement-I: $x = \sqrt[3]{-0.19683}$
Statement-II: $y = \sqrt[3]{x}$

65. Let a, b and c be the sides of a triangle ABC.
Question : Is the triangle equilateral ?
Statement-I: $a^2 + b^2 + c^2 = (ab + bc + ca)$
Statement-II: $3a^2 + 3b^2 + 4c^2 = 2ab + 4bc + 4ca$

66. Area of a rectangle with length x and breadth y is P and area of a parallelogram (which is strictly not a rectangle) with adjacent sides of length x and y is Q.
Question : Is P > Q?
Statement-I: $x : y = 2 : 1$
Statement-II: The angle between the two adjacent sides of the parallelogram is 60°.

67. A circle touches all the four sides AB, BC, CD, DA of a quadrilateral ABCD.
Question: What is the perimeter of the quadrilateral ?
Statement-I: AB + DC = 10 cm
Statement-II: AD + BC = 10 cm

68. What is the ratio of the lengths of diagonals of a rhombus?
Statement-I: One diagonal of the rhombus is equal to its side.
Statement-II: The longer diagonal of the rhombus is equal to $\sqrt{3}$ times its side.

69. The chord of a circle of radius R touches at a point on the circumference of a concentric circle of radius r. The length of the chord is 24 units.
Question : What are the values of r and R?
Statement-I: r is an integer.
Statement-II: R is an integer.

70. P, Q,R,S are the mid-points of sides AB, BC, CD, DA respectively of a quadrilateral ABCD.
Question : What is the difference in the area of the quadrilateral ABCD and the area of the quadrilateral PQRS ?
Statement-I: Area of the quadrilateral ABCD is 100 square unit.
Statement-II: Area of the quadrilateral PQRS is 50 square unit.

Consider the following for the next two (02) items that follow :

In a pie-diagram (with radius 7 cm), the central angles of the sectors are in the ratio 2 : 3 : 7 : 5 : 1,

(Take $\pi = \frac{22}{7}$)

71. If P is the area of the smallest sector and Q is the area of the largest sector, then what is P + Q equal to?

(a) $\frac{88}{3}$ square cm **(b)** $\frac{77}{3}$ square cm

(c) $\frac{149}{6}$ square cm **(d)** $\frac{616}{9}$ square cm

72. If p is the perimeter of the smallest sector, then what is the value of $9p$?

(a) 142 cm **(b)** 148 cm
(c) 156 cm **(d)** 221 cm

Consider the following for the next three (03) items that follow:

Two trains A and B started from stations P and Q respectively towards each other. Train A started at 7 p.m. at a speed of 60 km/h and train B started at 4 a.m. (next day) at a speed of 90 km/h. The distance between the two stations P and Q is 800 km.

73. How far from station Q will the two trains meet?

(a) 104 km **(b)** 144 km
(c) 156 km **(d)** 504 km

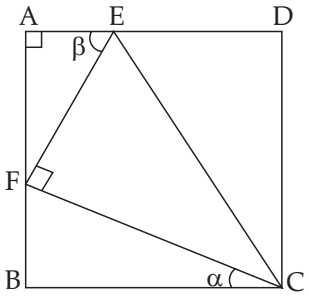
74. At what time will the two trains meet?

(a) 5 : 28 a.m. **(b)** 5 : 44 a.m.
(c) 4 : 56 a.m. **(d)** 6 : 24 a.m.

75. If the lengths of the two trains A and B are 400 m and 500 m respectively, then what is the time taken by them to cross each other?

(a) 21. 6 seconds **(b)** 18.2 seconds
(c) 17.4 seconds **(d)** 15.4 seconds

Consider the following for the next three (03) items that follow:
A triangle CEF is drawn inside a square ABCD as shown in the figure given below. Given: CF = 8 cm, EF = 6 cm and CE = 10 cm.



76. What is the area of the square?

(a) $\frac{512}{17}$ square cm **(b)** $\frac{625}{13}$ square cm

(c) $\frac{1024}{17}$ square cm **(d)** $\frac{1296}{13}$ square cm

77. What is $\tan\alpha + \tan\beta$ equal to?

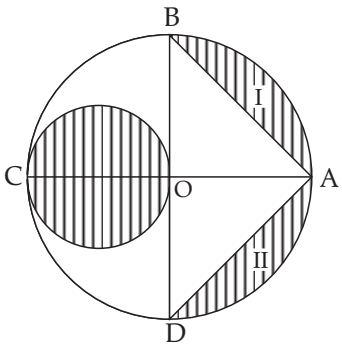
(a) $\frac{13}{16}$ **(b)** $\frac{15}{16}$ **(c)** $\frac{17}{16}$ **(d)** $\frac{17}{4}$

78. What is the area of triangle CDE ?

(a) $\frac{416}{17}$ square cm **(b)** $\frac{312}{13}$ square cm

(c) $\frac{208}{17}$ square cm **(d)** $\frac{156}{13}$ square cm

Consider the following for the next two (02) items that follow:
ABCD is a circle with centre O and taking OC as a diameter, a circle is drawn as shown in the figure given below. Let OB = 7 cm. $\left(\text{Use } \pi = \frac{22}{7}\right)$



79. What is the area of the shaded region?

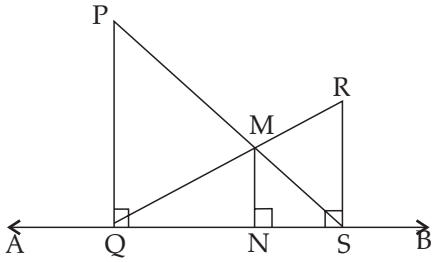
(a) 38.5 square cm **(b)** 48 square cm

(c) 52.5 square cm **(d)** 66.5 square cm

80. What is the ratio of the area of the shaded region to the area of the non-shaded region?

(a) $\frac{19}{25}$ **(b)** $\frac{18}{25}$ **(c)** $\frac{17}{25}$ **(d)** $\frac{16}{25}$

Consider the following for the next two (02) items that follow:
Let two parallel line segments PQ = 5 cm and RS = 3 cm be perpendicular to a horizontal line AB, as shown in the figure given below. The point of intersection of PS and QR is M and MN is perpendicular to QS.



81. What is the length of MN?

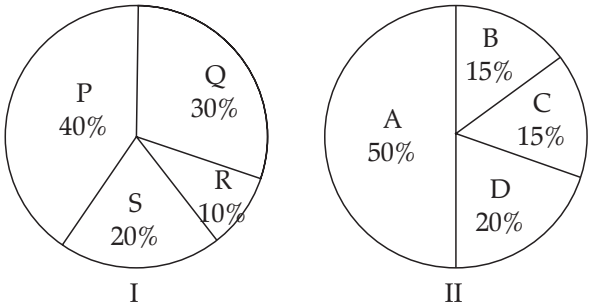
(a) $\frac{3}{8}$ cm (b) $\frac{5}{8}$ cm

(c) $\frac{9}{8}$ cm **(d)** $\frac{15}{8}$ cm

82. What is the ratio of the area of the quadrilateral PQNM to the area of the quadrilateral RSNM?

(a) $\frac{200}{117}$ **(b)** $\frac{212}{117}$ **(c)** $\frac{275}{117}$ **(d)** $\frac{250}{117}$

Consider the following for the next three (03) items that follow:
The following Pie-Chart-I shows the people migrating to Delhi from different Indian States (P, Q and R are three different States and S is the combined group of other States) and Pie-Chart-II indicates the different age groups A, B, C and D of these migrating people for each State.



83. If the people coming from a particular State belonging to S are 15% and 24,000 in number, then what is the total number of migrating people belonging to the age group B ?

(a) 1.2 lac **(b)** 1.25 lac

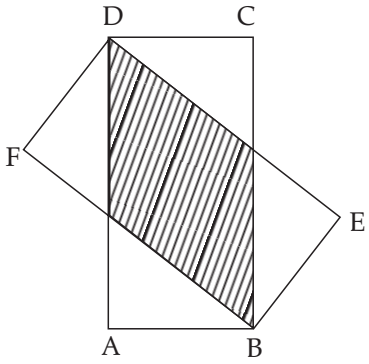
(c) 1.30 lac **(d)** 1.50 lac

84. What is the maximum of differences between the number of people coming from different groups P, Q, R and S?

(a) 1.6 lac **(b)** 1.8 lac **(c)** 2.4 lac **(d)** 2.6 lac

85. What is the difference between number of people coming from R having age group A and those coming from Q having age group D ?
(a) 6,000 **(b)** 8,000
(c) 12,000 **(d)** 18,000

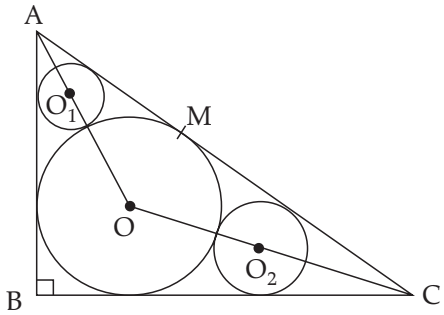
Consider the following for the next two (02) items that follow :
Consider two identical rectangles ABCD and BEDF as shown in the figure given below. Let AB = 1 cm and BC = 2 cm.



86. What is the area of the overlapping region?
(a) $\frac{8}{5}$ square cm **(b)** $\frac{5}{4}$ square cm
(c) $\frac{4}{5}$ square cm **(d)** $\frac{3}{4}$ square cm

87. What is the area of the non-overlapping region?
(a) $\frac{3}{4}$ square cm **(b)** $\frac{11}{4}$ square cm
(c) $\frac{3}{2}$ square cm **(d)** $\frac{5}{4}$ square cm

Consider the following for the next three (03) that follow:
ABC is a right-angled triangle with $\Delta ABC = 90°$. The centre of the incircle of the given triangle is at O, whose radius is 2 cm. Two more circles with centres at O_1 and O_2, touch this circle and the two sides as shown in the figure given below.
Further, MA : MC = 2 : 3.



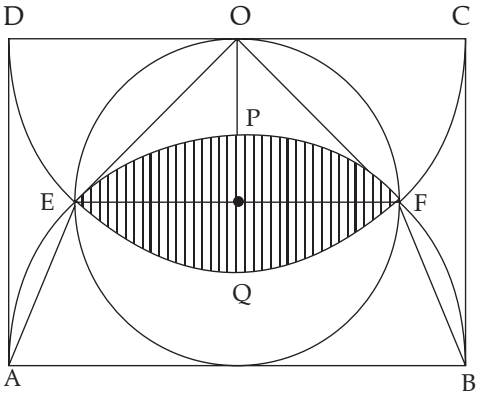
88. What is AB + BC equal to?
(a) 10 cm **(b)** 12 cm
(c) 13 cm **(d)** 14 cm

89. What is the radius of the circle with centre at O_1?
(a) $4-\sqrt{5}$ **(b)** $1+\sqrt{5}$
(c) $2+\sqrt{5}$ **(d)** $3-\sqrt{5}$

90. What is the radius of the circle with centre at O_2?
(a) $5-\sqrt{10}$ **(b)** $1+2\sqrt{5}$
(c) $\frac{22-4\sqrt{10}}{9}$ **(d)** $\frac{22-2\sqrt{10}}{9}$

Consider the following for the next three (03) items that follow:
Consider two identical semicircles and one circle inscribed in a rectangle of length 10 cm a shown in the figure given below.

(Take $\pi = 3.14$ and $\sqrt{2} = 1.4$)

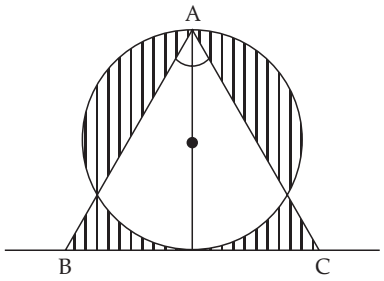


91. What is the area of triangle EOF ?
(a) $12.5\sqrt{3}$ sq. cm (b) $6.25\sqrt{3}$ sq. cm
(c) 12.5 sq. cm (d) 6.25 sq. cm

92. What is the area of trapezium AEFB?
(a) 30 square cm **(b)** 25 square cm
(c) 20 square cm **(d)** 18.75 square cm

93. What is the area of the shaded region?
(a) 14.75 square cm **(b)** 14.25 square cm
(c) 7.225 square cm **(d)** 7.625 square cm

Consider the following for the next two (02) items that follow:
Consider a circle of area 9π square unit and an equilateral triangle ABC as shown in the figure given below.



94. What is the length of the side of the triangle ABC?

(a) $2\sqrt{3}$ unit (b) $4\sqrt{3}$ unit

(c) $6\sqrt{3}$ unit **(d)** $8\sqrt{3}$ unit

95. What is the area of the shaded region?

(a) $6(\pi+\sqrt{3})$ square unit

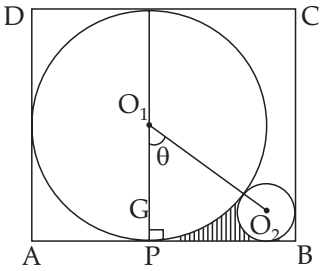
(b) $3(\pi+3\sqrt{3})$ square unit

(c) $1.5(3\pi+8\sqrt{3})$ square unit

(d) $6(\pi+2\sqrt{3})$ square unit

Consider the following for the next three (03) items that follow :

Two circles with centres at O_1 and O_2 touching each other are placed inside a rectangle of sides 9 cm and 8 cm as shown in the figure given below.



96. What is the sum of the areas of the two circles?

(a) 17 π square unit
(b) 16.75 π square unit
(c) 16.5 π square unit
(d) 16.25 π square unit

97. Which one of the following is correct in respect of angle θ?

(a) $0 < \theta < 30°$ **(b)** $30° < \theta < 45°$
(c) $45° < \theta < 60°$ **(d)** $60° < \theta < 90°$

98. What is the area of the shaded region?

(a) $\frac{240-10\pi-\pi\theta}{24}$ square unit

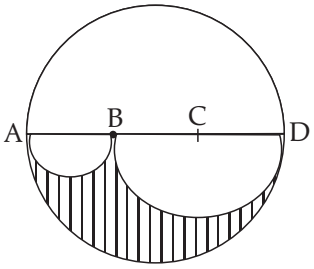
(b) $\frac{240-6\pi-\pi\theta}{24}$ square unit

(c) $\frac{120-12\pi-\pi\theta}{24}$ square unit

(d) $\frac{240-12\pi-\pi\theta}{24}$ square unit

Consider the following for the next two (02) items that follow:

Let ABCD be the diameter of a circle of radius 6 cm. The lengths AB, BC and CD are equal. Semi-circles are drawn with AB and BD as diameters as shown in the figure given below.



99. What is the ratio of the area of the shaded region to that of the non-shaded region?

(a) 2 : 7 **(b)** 2 : 5 **(c)** 3 : 5 **(d)** 5 : 8

100. What is the perimeter of the shaded region?

(a) 24π cm **(b)** 18π cm
(c) 15π cm **(d)** 12π cm

Answer Key

Q.No.	Answer	Topic's Name	Chapter's Name
1	a	Ratio	Number System
2	b	Speed	Speed, Distance & Time
3	a	Ratio	Ratio and Proportion
4	c	Simple and Compound Interest	Arithmetic
5	d	Gain Percentage	Arithmetic
6	d	Angles	Mensuration
7	c	Quadratic Equation	Quadratic Equation
8	b	Zeros of Polynomial	Polynomial
9	d	Use of trigonometric tables	Trigonometry
10	b	Zeros of Polynomial	Polynomial
11	b	Roots	Quadratic Equation
12	c	Greatest and Least Value	Trigonometry
13	b	Area	Geometry
14	b	Nature of roots	Quadratic Equation

Q.No.	Answer	Topic's Name	Chapter's Name
15	**b**	Mean, Median, Mode	Statistics
16	**a**	Division algorithm	Number theory
17	**c**	Remainder theorem	Number theory
18	**b**	Remainder theorem	Number theory
19	**b**	Remainder theorem	Number theory
20	**c**	Time and Work	Arithmetic
21	**c**	Cubes	Algebra
22	**b**	HCF	Number system
23	**a**	HCF	Number system
24	**a**	HCF	Number system
25	**d**	Divisibility	Number theory
26	**d**	Divisibilty	Number theory
27	**a**	Divisibilty	Number theory
28	**c**	AM, GM	Statistics
29	**b**	Time and Work	Number System
30	**a**	Operations	Number System
31	**d**	Mixture	Mixture
32	**a**	Trigonometric identities	Trigonometry
33	**a**	Remainder theorem	Number theory
34	**c**	Trigonometric identities	Trigonometry
35	**a**	Operations	Number System
36	**a**	Operations	Number System
37	**c**	Surface area	Mensuration
38	**b**	Volume	Mensuration
39	**b**	Circles	Geometry
40	**a**	Circles	Geometry
41	**b**	Triangles	Mensuration
42	**d**	GM	Statistics
43	**b**	Profit Percentage	Number System
44	**b**	Simple Interest	Number System
45	**a**	Time and Work	Number System
46	**c**	Ratio	Number System
47	**d**	Simple and Compound Interest	Number System
48	**b**	Quadrilateral	Geometry
49	**a**	Circles	Geometry
50	**c**	Quadrilateral	Geometry
51	**c**	Trigonometric Identities	Trigonometry
52	**c**	Trigonometric Identities	Trigonometry
53	**a**	Heights and Distances	Trigonometry
54	**c**	Trigonometric Identities	Trigonometry
55	**d**	Time and Distance	Number System
56	**c**	Volume	Mensuration
57	**a**	Ratio	Number System
58	**d**	Direct and Inverse proportions	Number System

Q.No.	Answer	Topic's Name	Chapter's Name
59	**c**	Time and Work	Number System
60	**d**	Minimum value	Number theory
61	**c**	Operations	Number System
62	**d**	Operations	Number System
63	**a**	Remainder Theorem	Number System
64	**c**	Operations	Number System
65	**b**	Triangles	Geometry
66	**c**	Area	Mensuration
67	**b**	Quadrilateral	Mensuration
68	**b**	Quadrilateral	Mensuration
69	**d**	Circles	Geometry
70	**b**	Quadrilateral	Mensuration
71	**d**	Circles	Geometry
72	**b**	Circles	Geometry
73	**c**	Time and Distance	Number System
74	**b**	Time and Distance	Number System
75	**a**	Time and Distance	Number System
76	**c**	Area	Mensuration
77	**d**	Angles	Mensuration
78	**a**	Area	Mensuration
79	**d**	Area	Mensuration
80	**a**	Area	Mensuration
81	**d**	Triangles	Geometry
82	**c**	Triangles	Geometry
83	**a**	Pie-Chart	Statistics
84	**c**	Pie-Chart	Statistics
85	**b**	Pie-Chart	Statistics
86	**b**	Area	Mensuration
87	**c**	Area	Mensuration
88	**d**	Triangles	Geometry
89	**d**	Circles	Geometry
90	**c**	Circles	Geometry
91	**c**	Area	Mensuration
92	**a**	Area	Mensuration
93	**b**	Area	Mensuration
94	**b**	Area of triangle	Mensuration
95	**b**	Area	Mensuration
96	**a**	Area of circle	Mensuration
97	**c**	Angles	Mensuration
98	**d**	Area	Mensuration
99	**a**	Area	Mensuration
100	**d**	Perimeter	Mensuration

C.D.S.
Combined Defence Services

EXAM
I
2024

Elementary Mathematics

ANSWERS WITH EXPLANATION

1. Option (a) is correct.

Explanation: Given:

$a : b : c : d = \sqrt{4} : \sqrt{3} : \sqrt{2} : \sqrt{1}$

$= 2 : \sqrt{3} : \sqrt{2} : 1$

Let $a = 2k,\ b = \sqrt{3}k, c = \sqrt{2}k$ and $d = k$

So, $\dfrac{-a^2 + b^2 + c^2 + d^2}{a^2 - b^2 + c^2 - d^2}$

$= \dfrac{-4k^2 + 3k^2 + 2k^2 + k^2}{4k^2 - 3k^2 + 2k^2 - k^2} = \dfrac{2k^2}{2k^2} = 1$

2. Option (b) is correct.

Explanation: Since, speed $= \dfrac{\text{distance}}{\text{time}}$

So, according to the question, we have

$(2u \times xt) = (3u \times 4t) = (4u \times 3t) = (xu \times 2t)$

$\Rightarrow 2uxt = 12ut$

$\Rightarrow x = 6$

3. Option (a) is correct.

Explanation: Since $m : n = 1 : 2 \Rightarrow m = \dfrac{n}{2}$

and $p : q = 3 : 4 \Rightarrow p = \dfrac{3q}{4}$

So, $\dfrac{2m + 4p}{n + 3q} = \dfrac{2\left(\frac{n}{2}\right) + 4\left(\frac{3q}{4}\right)}{n + 3q} = \dfrac{n + 3q}{n + 3q} = \dfrac{1}{1}$

i.e, $(2m + 4p) \,.\, (n + 3q) = 1 : 1$

4. Option (c) is correct.

Explanation: Principal (P) = ₹10,000

Rate of interest

(R) = 5% p.a., Time (T) = 3 years

So, C.I. – S.I. $= \mathrm{P}\left(1 + \dfrac{\mathrm{R}}{100}\right)^{\mathrm{T}} - \mathrm{P} - \dfrac{\mathrm{P} \times \mathrm{R} \times \mathrm{T}}{100}$

$= 10,000\left\{\left(1 + \dfrac{5}{100}\right)^3 - 1 - \dfrac{5 \times 3}{100}\right\}$

$= 10,000\left\{\dfrac{105 \times 105 \times 105 - (100)^3 - 15 \times (100)^2}{10,00,000}\right\}$

$= \dfrac{1}{100}\{7,625\} = ₹\ 76.26$

Short-cut: 3 year C.I. and S.I. difference

$= \dfrac{\mathrm{PR}^2(300 + \mathrm{R})}{(100)^3} = \dfrac{10,000 \times 5 \times 5 \times (300 + 5)}{(100)^3}$

$= \dfrac{305}{40} = 76.26$

5. Option (d) is correct.

Explanation: Let listed price = ₹ x

So, C.P. $= \dfrac{3}{4} \times x = ₹\ \dfrac{3x}{4}$

S.P. $= x \times \dfrac{150}{100} = \dfrac{3x}{2}$

$\therefore$ Gain percentage $= \dfrac{\text{SP} - \text{C.P.}}{\text{C.P.}} \times 100$

$= \dfrac{\frac{3}{2}x - \frac{3}{4}x}{\frac{3}{4}x} \times 100$

$= \dfrac{3}{4} \times \dfrac{4}{3} \times 100 = 100\%$

6. Option (d) is correct.

Explanation: Let interior angle = x and exterior angle = y

So, $x + y = 180°$ ...(i)

and $x - y = 144°$ ...(ii)

On solving (i) and (ii), we get

$2x = 324° \Rightarrow x = 162°$

and $y = 18°$

So, number of sides of the regular polygon

$= \dfrac{360°}{18°} = 20$

7. Option (c) is correct.
Explanation: Sum of roots = 2;
Product of roots = –100
So, required quadratic equation is
x^2 – (Sum of roots) x + product of roots = 0
$x^2 - 2x - 100 = 0$

8. Option (b) is correct.
Explanation: Here, $p(x) = x^3 + 3x^2 - 6x - a$
$\because\ p(2) = 0$
$\Rightarrow (2)^3 + 3(2)^2 - 6\,(2) - a = 0$
$\Rightarrow 8 + 12 - 12 - a = 0$
$\Rightarrow a = 8$
Now, $x - 2$ is a factor of $p(x) = x^3 + 3x^2 - 6x - 8$

$$
\begin{array}{r|l}
 & x^2 + 5x + 4 \\
\hline
x - 2 & x^3 - 3x^2 - 6x - 8 \\
 & x^3 - 2x^2 \\
 & - \quad + \\
\hline
 & 5x^2 - 6x \\
 & 5x^2 - 10x \\
 & - \quad + \\
\hline
 & 4x - 8 \\
 & 4x - 8 \\
 & - \quad + \\
\hline
 & 0
\end{array}
$$

$x^2 + 5x + 4 = x^2 + 4x + x + 4$
$= x(x + 4) + 1\,(x + 4) = (x + 1)\,(x + 4)$
So, other roots are : –1 and –4
$\therefore$ Required value = $(-1)^2 + (-4)^2$
$= 1 + 16 = 17$

9. Option (d) is correct.
Explanation: Given: $t = \cos 79°$

$\sin 79° = \sqrt{1 - \cos^2 79°} = \sqrt{1 - t^2}$

So, cosec 79° (1 – cos79°)

$$= \frac{1}{\sin 79°}(1 - \cos 79°)$$

$$= \frac{1-t}{\sqrt{1-t^2}} = \frac{\sqrt{1-t}\sqrt{1-t}}{\sqrt{1-t}\sqrt{1+t}} = \sqrt{\frac{1-t}{1+t}}$$

10. Option (b) is correct.
Explanation: Since, α, β, γ and δ are zeroes of $p(x)$
So, $p(x) = (x - \alpha)\,(x - \beta)(x - \gamma)(x - \delta)$
Also, α, β, γ, λ are zeroes of $q(x)$
So, $q(x) = (x - \alpha)\,(x - \beta)\,(x - \gamma)(x - \lambda)$

$$\frac{p(x) - q(x)}{(x-\alpha)(x-\beta)(x-\gamma)}$$

$$= \frac{(x-\alpha)(x-\beta)(x-\gamma)(x-\delta-x+\lambda)}{(x-\alpha)(x-\beta)(x-\gamma)}$$

$= \lambda - \delta$

11. Option (b) is correct.
Explanation: Since, $x\cos\theta = x^2 + p$ has a real solution
So, $x^2 - x\cos\theta + p = 0$ has a real solution
So, $D \geq 0$
$\Rightarrow (-\cos\theta)^2 - 4\,(1)\,(p) \geq 0$
$\Rightarrow \cos^2\theta - 4p \geq 0$
$\Rightarrow \cos^2\theta \geq 4p$

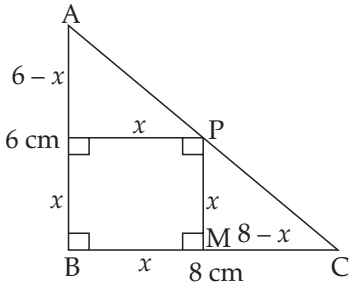
$$\Rightarrow 4p \leq \cos^2\frac{\pi}{4} = \frac{1}{2}$$

$$\Rightarrow p \leq \frac{1}{8}$$

12. Option (c) is correct.
Explanation: $\cos^2\theta + 3\sin^2\theta + 2$
$= \cos^2\theta + \sin^2\theta + 2\sin^2\theta + 2$
$= 1 + 2\sin^2\theta + 2 \Rightarrow 2\sin^2\theta + 3$
$-1 \leq \sin\theta \leq 1$
$\Rightarrow 0 \leq \sin^2\theta \leq 1$
$\Rightarrow 0 \leq 2\sin^2\theta \leq 2$
$\Rightarrow 3 \leq 2\sin^2\theta + 3 \leq 5$
$\therefore$ least value = 3 and greatest value = 5
So, required difference = 5 – 3 = 2
Short-cut:
Least value = $\cos^2\theta + 3\sin^2\theta + 2$ $(\theta = 0^o)$
= 3
Max. value = $\cos^2 90 + 3\sin^2 90 + 2$ $(\theta = 90^o)$
= 5
Difference = 5 – 3 = 2

13. Option (b) is correct.
Explanation: Using pythagoras, theorem,

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{6^2 + 8^2} = 10 \text{ cm}$$



$\Delta ABC \sim \Delta PMC$

$$\therefore \frac{BC}{MC} = \frac{AB}{PM}$$

$$\Rightarrow \frac{8}{8-x} = \frac{6}{x} \Rightarrow 8x = 48 - 6x$$

$$\Rightarrow 14x = 48 \Rightarrow x = \frac{48}{14} = \frac{24}{7} \text{ cm}$$

$$\therefore \text{Perimeter of square} = 4 \times \frac{24}{7} = \frac{96}{7} \text{ cm}$$

14. Option (b) is correct.

Explanation: $2x^2 - 4x + k = 0$ has real roots

So, $D \geq 0$

$(-4)^2 - 4(2)(k) \geq 0$

$\Rightarrow 16 - 8k \geq 0$

$\Rightarrow 16 \geq 8k \Rightarrow k \leq 2$

So, maximum value of $k = 2$

15. Option (b) is correct.

Explanation: Given data in ascending order is:

36, 41, 43, 61, 71, 92, 93, 95, 99, 110, 110, 127

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Total no. of observations}}$$

$$= \frac{978}{12} = 81.5$$

$$\text{Median} = \frac{\left(\left(\frac{12}{2}\right)^{\text{th}} + \left(\frac{12}{2}+1\right)^{\text{th}}\right)}{2} \text{ observation}$$

$$= \frac{(6^{\text{th}} + 7^{\text{th}}) \text{ observation}}{2}$$

$$= \frac{92+93}{2} = \frac{185}{2} = 92.5$$

Mode = 110

If 93 is replaced with 94, so the data is :

36, 41, 43, 61, 71, 92, 94, 95, 99, 110, 110, 127

$$\text{New mean} = \frac{978 - 93 + 94}{12} = \frac{979}{12} = 81.6 \text{ (approx)}$$

$$\text{New median} = \frac{(6^{\text{th}} + 7^{\text{th}})}{2} \text{observation}$$

$$= \frac{92+94}{2} = \frac{186}{2} = 93$$

New mode = 110

1. Difference = 93 – 92.5 = 0.5
2. Difference = 81.6 – 81.5 = 0.1
3. Difference = 110 – 110 = 0

16. Option (a) is correct.

Explanation: $\frac{(225)^{40}}{1000} = \frac{5^{80} \times 9^{40}}{1000} = \frac{5^{77} \times 9^{40}}{8}$

$\frac{9^{40}}{8} \rightarrow$ Remainder 1

$\frac{5^{77}}{8} \rightarrow$ Remainder 5

So, last three digits = $125 \times 5 = 625$

$\therefore$ Hundredths place is 6

17. Option (c) is correct.

Explanation: Unit digits of

$1^a \rightarrow 1$,

$2^b \rightarrow 2, 4, 6, 8, ...$

$3^c \rightarrow 3, 9, 7, 1, ...$

$4^d \rightarrow 4, 6, ...$

So, possible remainders are : 0, 2, 4, 6 and 8

18. Option (b) is correct.

Explanation: Let $n = 3$,

$9^3 + 11^3 = 729 + 1331 = 2060$, when divided by 10

remainder = 0

Let $n = 2$

$9^2 + 11^2 = 202$, when divided by 10, remainder = 2

Now, $6^3 + 4^3 = 280$, when divided by 10, remainder = 0

Let $n = 4$

$6^4 + 4^4 = 1552$, when divided by 10, remainder = 2

So, required sum = 2 + 2 = 4

19. Option (b) is correct.

Explanation: Last four digits = 9100, which when divided by 16, remainder = 12

20. Option (c) is correct.

Explanation: Since, efficiency of A and B is more than the others.

Amount of work done by A and B in 2 hours

$$= \frac{1}{3} + \frac{1}{6} = \frac{2+1}{6} = \frac{1}{2}$$

So, required number of hours to complete the work = $2 \times 2 = 4$

21. Option (c) is correct.

Explanation: $p = \sqrt[3]{(a + \sqrt{a^2 + b^3})} + \sqrt[3]{a - (a^2 + b^3)}$

On cubing both sides, we get

$$p^3 = (a + \sqrt{a^2 + b^3}) + (a - \sqrt{a^2 + b^3}) + 3\left(\sqrt[3]{a^2 - (a^2 + b^3)}\right)\left(\sqrt[3]{a + \sqrt{a^2 + b^3}} + \sqrt[3]{a - (a^2 + b^3)}\right)$$

$\Rightarrow p^3 = 2a + 3(-b)\, p$

$\Rightarrow p^3 + 3bp = 2a$

22. Option (b) is correct.

Explanation: Length of wood = 4.25 m = 425 cm

Width of wood = 3.4 m = 340 cm

$\therefore$ HCF (425, 340) = 85 cm

$$\therefore \text{Number of pieces} = \frac{425 \times 340}{85 \times 85} = 20$$

23. Option (a) is correct.

Explanation: Put $x = 2$ and $y = 1$

$x^4 - 13x^2y^2 - 300y^4$

$= 16 - 13(4) - 300 = 16 - 52 - 300 = -336$

$x^3 - 4x^2y - 4xy^2 - 5y^3 = 8 - 16 - 8 - 5 = -21$
$x^3 - 125y^3 = 8 - 125 = -117$
So, H.C.F. $= -3$
i.e., $x - 5y$

24. Option (a) is correct.
Explanation: Let $x = 2$ and $y = 3$
$\therefore$ HCF (768, 576) = 192 = 32 (2) (3)
$\therefore x + y = 2 + 3 = 5$

25. Option (d) is correct.
Explanation: As, $910 = 7 \times 13 \times 10$
$\therefore$ Smallest value of n such that $(n + 1)!$ as divisible by 13 is 12.
So, required value = 12

26. Option (d) is correct.
Explanation: The given expression is divisible by 2, 3 and 37.

27. Option (a) is correct.

Explanation: **1.** $\dfrac{3m^2 + 2m^2 + 5m + n}{m}$

$= \underbrace{3m + 2m + 5}_{\text{integer}} + \dfrac{n}{m}$

So, statement-1 is correct.
2. For $m = 0$,
$5(8^0) + 2^0 = 5 + 1 = 6$, not divisible by 48
So, statement-2 is not correct.

28. Option (c) is correct.
Explanation: Let two positive numbers of
$\therefore$ According to the question,

$$\sqrt{(a)(40-a)} = \frac{a+40-a}{2} - \frac{20}{100}\left[\frac{a+40-a}{2}\right]$$

$$\Rightarrow \sqrt{a(40-a)} = 20 - 4 = 16$$

$\Rightarrow a(40 - a) = 256$
$\Rightarrow a^2 - 40a + 256 = 0$
$\Rightarrow (a - 32)(a - 8) = 0$
$\therefore a = 8, 32$
So, required difference $= 32 - 8 = 24$

29. Option (b) is correct.

Explanation: 1 man's 1 day work $= \dfrac{1}{50 \times 40}$

Total work done $= 50 \times 40 = 2000$ units
Amount of work done in 1st 10 days
$= 50 \times 10 = 500$ units
Amount of work done in 2nd 10 days
$= 45 \times 10 = 450$ units
Amount of work done in 3rd 10 days
$= 40 \times 10 = 400$ units
Amount of work done in 4th 10 days
$= 35 \times 10 = 350$ units
Amount of work done in 5th 10 days
$= 30 \times 10 = 300$ units
Total work done
$= 500 + 450 + 400 + 350 + 300 = 2000$
$\therefore$ No. of days = 50

30. Option (a) is correct.

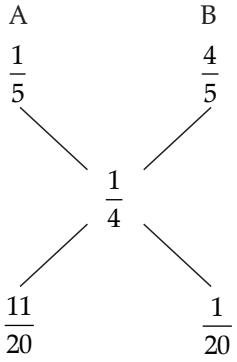
Explanation: $x = \cfrac{1}{2 + \cfrac{3}{4 + \cfrac{5}{6 + \cfrac{7}{8 + \cfrac{9}{10}}}}}$

$$= \frac{1}{2 + \dfrac{3}{4 + \dfrac{5}{6 + \dfrac{70}{89}}}} = \frac{1}{2 + \dfrac{3}{4 + \dfrac{445}{604}}}$$

$$= \frac{1}{2 + \dfrac{1812}{2861}} = \frac{2861}{7534} = 0.38$$

So, $0 < x < 0.5$

31. Option (d) is correct.
Explanation: Using allegation method,



So, required ratio = 11 : 1

32. Option (a) is correct.
Explanation: Given : $3\sin\theta + 5\cos\theta = 5$
$\Rightarrow 9\sin^2\theta + 25\cos^2\theta + 30\sin\theta\cos\theta = 25$ ... (i)
Now, let $5\sin\theta - 3\cos\theta = x$
$\Rightarrow 25\sin^2\theta + 9\cos^2\theta - 30\sin\theta\cos\theta = x^2$ ...(ii)
Adding (i) and (ii)
$9(\sin^2 + \cos^2\theta) + 25(\sin^2\theta + \cos^2\theta) = 25 + x^2$
$\Rightarrow 9 = x^2 \Rightarrow x = \pm 3$
Short-cut : Put $\theta = 0$
$3\sin\theta + 5\cos\theta = 5$
Now, $5\sin\theta - 3\cos\theta = -3$

33. Option (a) is correct.
Explanation:
Let $p(x) = x^{4k} + x^{4k+2} + x^{4k+4} + x^{4k+6}$
1. $x^2 + 1 = 0 \Rightarrow x^2 = -1$

So, $p(x) = (x^2)^{2k} + (x^2)^{2k+1} + (x^2)^{2k+2} + (x^2)^{2k+3}$
Remainder $= 1 - 1 + 1 - 1 = 0$
So, statement 1 is true

2. $x^4 = -1$

So, $p(x) = (x^4)^{2k} + (x^4)^k x^2 + (x^4)^{k+1} + (x^4)^k . x^6$

So, statement-2 is false, as it depends on value of k.

34. Option (c) is correct.

Explanation:

$$\frac{\sin^2 A + 5\sin A + 1}{\sin A} = \sin A + 5 + \frac{1}{\sin A}$$

$\because x + \frac{1}{x} \geq 2$ if $x \geq 0$

$\therefore$ Minimum value = 2 + 5 = 7

35. Option (a) is correct.

Explanation: $\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + ...$

$$= \frac{2^2 - 1^2}{1^2 \times 2^2} + \frac{3^2 - 2^2}{2^2 \times 3^2} + ...$$

$$= \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{2^2} - \frac{1}{3^2} + ...$$

$$= \frac{1}{1} = 1$$

36. Option (a) is correct.

Explanation: Given: $\cfrac{1}{a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d + \cfrac{1}{e}}}}} = \frac{421}{972}$

$\because \frac{972}{421} = 2 + \frac{130}{421}$

$\therefore$ We have, $a = 2$

Now, $\frac{421}{130} = 3 + \frac{31}{130} \Rightarrow b = 3$

Now, $\frac{130}{31} = 4 + \frac{6}{31} \Rightarrow c = 4$

Now, $\frac{31}{6} = 5 + \frac{1}{6} \Rightarrow d = 5$

Now, $e = 6$

$\therefore a \times b \times c \times d \times e = 2 \times 3 \times 4 \times 5 \times 6 = 720$

37. Option (c) is correct.

Explanation: Edge of cube = 14 cm

Radius of circle = 7 cm

So, area of 6 circles = $6 \times \pi \times r^2$

$$= 6 \times \frac{22}{7} \times 7 \times 7 = 924 \text{ cm}^2$$

$\therefore$ Total area of unpainted surface

= Total surface area of cube – Area of 6 circles

= $6 \times (14)^2 - 924 = 1176 - 924$

= 252 square cm

38. Option (b) is correct.

Explanation: According to the question,

Volume of remaining metal plate

= Volume of sphere

$$\Rightarrow \left[\frac{360° - 150°}{360°}\right] \times \frac{22}{7} \times (7)^2 \times \frac{16}{1000}$$

$$= \frac{4}{3} \times \frac{22}{7} \times r^3$$

$$\Rightarrow \frac{7}{12} \times 49 \times \frac{16}{1000} = \frac{4}{3} \times r^3$$

$$\Rightarrow r^3 = \frac{7 \times 49 \times 16 \times 3}{12 \times 4 \times 1000} = \frac{(7)^3}{(10)^3}$$

$\Rightarrow r = 0.7$ cm

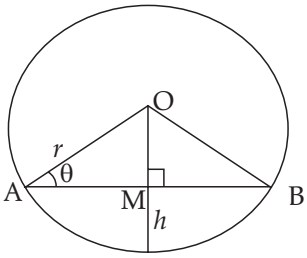
39. Option (b) is correct.

Explanation: AB = $2\sqrt{3}h$

OM = $(r - h)$

AM = $\sqrt{3}h$

In ΔAOM



$(OA)^2 = OM^2 + (AM)^2$

(By pythagoras theorem)

$\Rightarrow r^2 = (r - h)^2 + (\sqrt{3}h)^2$

$\Rightarrow r^2 = r^2 + h^2 - 2rh + 3h^2$

$\Rightarrow 2rh = 4h^2 \Rightarrow 2h(r - 2h) = 0$

$\Rightarrow r = 2h$

$\therefore \tan\theta = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30°$

$\therefore \angle AOM = 60° \Rightarrow \angle AOB = 120°$

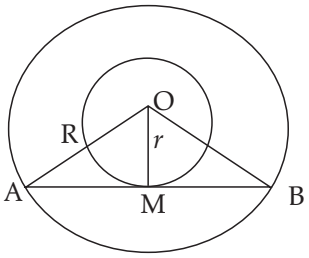
Now, $\frac{P}{Q} = \cfrac{\pi \times 4h^2 \times \frac{120°}{360°}}{\pi \times 4h^2 \times \frac{120°}{360°} - \frac{1}{2} \times 2\sqrt{3}h \times h}$

$$= \frac{4\frac{\pi}{3}}{\frac{4\pi}{3} - \sqrt{3}} = \frac{4\pi}{3} \times \frac{3}{4\pi - 3\sqrt{3}} = \frac{4\pi}{4\pi - 3\sqrt{3}} \simeq 1.7$$

40. Option (a) is correct.

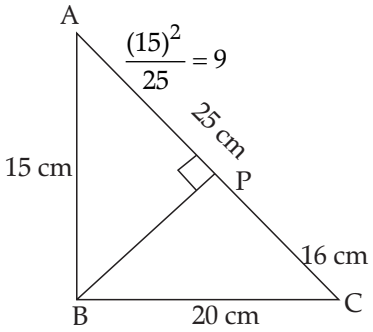
Explanation: Here, AB = 14 cm

$\therefore$ AM = MB = 7 cm



Using Pythagoras theorem,
$OA^2 = OM^2 + AM^2$
$\Rightarrow R^2 = r^2 + 7^2$
$\Rightarrow R^2 - r^2 = 49$
Now, area between two circles
$= \pi R^2 - \pi r^2 = \pi (R^2 - r^2)$
$= 49\pi = 49 \times \frac{22}{7}$
$= 154$ square cm.

41. Option (b) is correct.
Explanation: Area of ΔABC
$= \frac{1}{2} \times 20 \times 15 = \frac{1}{2} \times 25 \times BP$
$\Rightarrow \frac{20 \times 15}{25} = BP$
$\Rightarrow BP = 12$ cm



Required difference
$= |\text{area of } \Delta PAB - \text{area of } \Delta PCB|$
$= \left|\frac{1}{2} \times BP \times AP - \frac{1}{2} \times BP \times PC\right|$
$= \frac{1}{2} \times 12 |(AP - PC)| = 6 |(AP - PC)|$
$= 6[16 - 9] = 42$ sq. cm

42. Option (d) is correct.
Explanation: $P = G.M. = \sqrt[n]{a_1 \times a_2 \times .... \times a_n}$
$\Rightarrow P^n = a_1 \times a_2 .... \times a_n$
$Q = GM_1 = \sqrt[2n]{a_{n+1} \times a_{n+2} \times .... \times a_{3n}}$
$\Rightarrow Q^{2n} = a_{n+1} \times .... a_{8n}$
Required G.M. $= (a_1 \times a_2 \times .... \times a_{3n})^{1/3n}$
$= \left[P^n \times Q^{2n}\right]^{\frac{1}{3n}} = P^{\frac{1}{3}} \times Q^{\frac{2}{3}}$

43. Option (b) is correct.
Explanation: Cost price of y articles = selling price z articles
So, profit% $= \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$
$= \frac{\text{S.P.}}{\text{C.P.}} - 1 \times 100$
$= \left(\frac{y}{z} - 1\right) \times 100 = \left(\frac{5}{4} - 1\right) \times 100$
$= \frac{1}{4} \times 100 = 25\%$

44. Option (b) is correct.
Explanation: Let principal = ₹ P
According to the question,
$(3P - P) = \frac{P \times R \times 8}{100}$
$\Rightarrow \frac{2 \times 100}{8} = R \Rightarrow R = 25\%$
So, $(nP - P) = \frac{P \times 25 \times 20}{100}$
$\Rightarrow \quad (n - 1) = 5$
$\Rightarrow \quad n = 6$

45. Option (a) is correct.
Explanation: According to the question,
$(x) \times (x + 1) = (x + 5)(x - 2)$
$\Rightarrow x^2 + x = x^2 + 3x - 10$
$\Rightarrow 10 = 2x \Rightarrow x = 5$

46. Option (c) is correct.
Explanation: Given : $(a + b) : (b + c) : (c + a)$
$= 5 : 7 : 6$
$\Rightarrow \frac{a+b}{b+c} = \frac{5}{7} \Rightarrow 7a + 7b = 5b + 5c$
$\Rightarrow 7a + 2b = 5c$ ...(i)
and $\frac{b+c}{c+a} = \frac{7}{6} \Rightarrow 6b + 6c = 7c + 7a$
$\Rightarrow 6b - c = 7a$ ...(ii)
$\frac{a+b}{c+a} = \frac{5}{6} \Rightarrow 6a + 6b = 5c + 5a$
$\Rightarrow a + 6b = 5c$ ...(iii)
Using (i) and (ii),
$6b - c + 2b = 5c \Rightarrow 8b = 6c$
$\Rightarrow 4b = 3c \Rightarrow \frac{b}{c} = \frac{3}{4} = \frac{6}{8}$
Using (i) and (iii)
$7a + 2b = a + 6b \Rightarrow 6a = 4b$
$\Rightarrow 3a = 2b \Rightarrow \frac{a}{b} = \frac{2}{3} = \frac{4}{6}$
$\therefore a : b : c = 4 : 6 : 8$

$$\frac{a-b+c}{a+b-c}=\frac{4k-6k+8k}{4k+6k-8k}=\frac{6k}{2k}=\frac{3}{1}$$

Short-cut: Let $a = 2, b = 3, c = 4$

$$\frac{a-b+c}{a+b-c}=\frac{2-3+4}{2+3-4}=\frac{3}{1}$$

47. Option (d) is correct.

Explanation: $x=\mathrm{P}\left[1+\frac{\mathrm{R}}{100}\right]^n-\mathrm{P}$

$$=1000\left[1+\frac{10}{100}\right]^3-1000$$

$$=1000\left\{\frac{1331}{1000}-1\right\}=₹\,331$$

and $y=\frac{\mathrm{P}\times\mathrm{R}\times\mathrm{T}}{100}=\frac{1000\times3\times11}{100}=₹\,330$

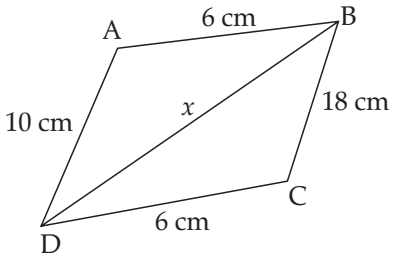
So, $x - y = ₹\,(331 - 330) = ₹\,1$

48. Option (b) is correct.

Explanation: Using ΔABD,

$(10 - 6)$ cm $< x <$ $(10 + 6)$ cm

$\Rightarrow 4 < x < 16$ ...(i)



Using ΔBCD,

$18 - 6 < x < 18 + 6$

$12 < x < 24$ ...(ii)

Using (i) and (ii)

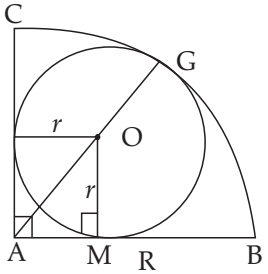
$12 < x < 16$

49. Option (a) is correct.

Explanation: Since, OM ⊥ AB

Here, AG = R

and AO = $\sqrt{r^2+r^2}=\sqrt{2}r$



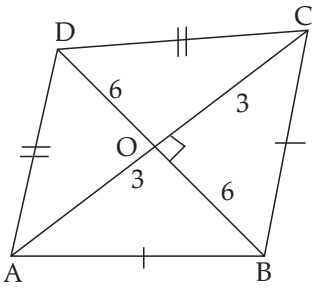
$\therefore$ AG = AO + OG = $\sqrt{2}r+r$

$\Rightarrow \mathrm{R} = r(\sqrt{2}+1)$

$$\Rightarrow \frac{\mathrm{R}}{r}=\frac{\sqrt{2}+1}{1}$$

50. Option (c) is correct.

Explanation: AC = 6 cm, BD = 12 cm



Area of quad. = area of (ΔABC) + area of (ΔACD)

$$=\frac{1}{2}\times\mathrm{AC}\times6+\frac{1}{2}\times\mathrm{AC}\times6$$

$$=2\times\frac{}{2}\times6\times6$$

= 36 sq. cm

51. Option (c) is correct.

Explanation: $\tan(3A) = \cot(A - 22°)$

$= \tan[90° - (A - 22°)] = \tan(112° - A°)$

$\Rightarrow 3A = 112° - A°$

$\Rightarrow 4A = 112° \Rightarrow A = 28°$

52. Option (c) is correct.

Explanation: Given :

$$\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=p\sec\theta+q\tan\theta$$

$$\Rightarrow\frac{\sin\theta-(\cos\theta-1)}{\sin\theta+(\cos\theta-1)}\times\frac{\sin\theta-(\cos\theta-1)}{\sin\theta-(\cos\theta-1)}$$

$$=\frac{\sin^2\theta+(\cos\theta-1)^2-2\sin\theta(\cos\theta-1)}{\sin^2\theta-(\cos\theta-1)^2}$$

$$=\frac{\sin^2\theta+\cos^2\theta+1-2\cos\theta+2\sin\theta(1-\cos\theta)}{\sin^2\theta-\cos^2\theta-1+2\cos\theta}$$

$$=\frac{2(1-\cos\theta)+2\sin\theta(1-\cos\theta)}{2\cos\theta[1-\cos\theta]}$$

$$=\frac{2(1+\sin\theta)}{2\cos\theta}=\sec\theta+\tan\theta$$

So, $p = 1$ and $q = 1$

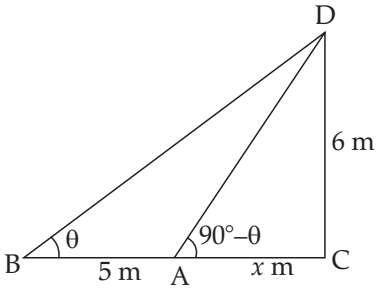
$\therefore p + q = 1 + 1 = 2$

53. Option (a) is correct.

Explanation: Here, CD = height of tower = 6 m

In ΔACD,

$$\frac{\mathrm{CD}}{\mathrm{AC}}=\tan(90°-\theta)$$



$\Rightarrow \frac{6}{x} = \cot\theta = \frac{1}{\tan\theta}$

$\Rightarrow \tan\theta = \frac{x}{6}$ ...(i)

In ΔBCD, $\frac{CD}{BC} = \tan\theta$

$\Rightarrow \frac{6}{x+5} = \tan\theta$ ...(ii)

Using (i) and (ii), we have

$\frac{x}{6} = \frac{6}{x+5}$

$\Rightarrow x^2 + 5x = 36$

$\Rightarrow x^2 + 5x - 36 = 0$

$\Rightarrow x^2 + 9x - 4x - 36 = 0$

$\Rightarrow x(x+9) - 4(x+9) = 0 \Rightarrow (x-4)(x+9) = 0$

$\therefore x = 4$ m

54. Option (c) is correct.

Explanation:

(1) If $A = B = C = \frac{\pi}{3}$

So, $\sin A + \sin B + \sin C = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$

$= \frac{3\sqrt{3}}{2}$

(2) If $A = B = C = \frac{\pi}{3}$

So, $\cos A + \cos B + \cos C = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$

55. Option (d) is correct.

Explanation: Speed of train A = 80 km/h

Speed of train B = 100 km/h

$\text{Time} = \frac{80 \times \frac{50}{60}}{\text{Rel. speed}} = \frac{400}{6 \times (100-80)} = \frac{20}{6} = \frac{10}{3}$

$\therefore$ Required distance $= 100\left\{\frac{10}{3}\right\} = \frac{1,000}{3}$ km

56. Option (c) is correct.

Explanation: Let length, breadth and height are : l, b and h respectively.

Volume of cuboid = $l\,b\,h$

New dimensions are:

$l_1 = \frac{110}{100} l = \frac{11}{10} l$

$b_1 = \frac{120}{100} b = \frac{12}{10} b$

$h_1 = \frac{150}{100} h = \frac{15}{10} h$

$\therefore$ New volume of cuboid $= \frac{11 \times 12 \times 15}{1000} l\,b\,h$

Now, required % increase

$= \frac{\left[\frac{11 \times 12 \times 15}{1000} - 1\right] lbh}{lbh} \times 100$

$= \frac{1,980 - 1,000}{1,000} \times 100 = \frac{980}{10} = 98\%$

57. Option (a) is correct.

Explanation: According to the question, we have

P'share = $3x + 93$, Q's share = $4x + 24$

and R's share = $5x + 55$

$\therefore 3x + 93 + 4x + 24 + 5x + 55 = 9,400$

$\Rightarrow 12x + 172 = 9,400$

$\Rightarrow 12x = 9,228$

$\Rightarrow x = 769$

$\therefore$ P'share = 3 (769)

= ₹ 2,307

58. Option (d) is correct.

Explanation: P^2 varies $R \Rightarrow P^2 \propto R \Rightarrow P^2 = kR$

Q^2 varies as $R \Rightarrow Q^2 \propto R \Rightarrow Q^2 = k'R$

1. $P^2 + Q^2 = (k + k')R \propto R$

True

2. $PQ = \sqrt{kR} \times \sqrt{k'R} = \sqrt{kk'}R \propto R$

True

3. $P^2 - Q^2 = (k - k')R$

$P^2 - Q^2 \propto R$

True

59. Option (c) is correct.

Explanation: Since, P men can finish work in q days

So, total work done = pq

According to the question,

$pq = \frac{150}{100} p(q-12)$

$\Rightarrow pq = \frac{3}{2} p(q-12)$

$\Rightarrow 2q = 3q - 36 \Rightarrow q = 36$

60. Option (d) is correct.

Explanation: $\left(\frac{a^2+3a+1}{a}\right)\left(\frac{b^2+3b+1}{b}\right)$

$= \left(a+3+\frac{1}{a}\right)\left(b+3+\frac{1}{b}\right)$

So, minimum value of $a+\frac{1}{a}=2$

$\therefore$ Required minimum value = (2 + 3) (2 + 3)
= 25

Short-cut: $a = 1, b = 1$

Now, $\left(\frac{1^2+3\times1+1}{1}\right)\cdot\left(\frac{1+3\times1+1}{1}\right) = 25$

61. Option (c) is correct.
Explanation: Given: $a > b > c$
and as c is not the smallest
So, we have $a > b > c > d$
$\therefore$ In the product of a, b, c, d; a is the closest to the product.

62. Option (d) is correct.
Explanation: Given, $mn = k$, where m, n are prime no. and k is an even no.
Since, k is even, so either m or n must be 2.
So, neither of the two statements will gives the answer.

63. Option (a) is correct.
Explanation: Statement I: If n is even.
P^n when divided by P + 1 gives remainder as 1.

64. Option (c) is correct.
Explanation: St. I: $x = \sqrt[3]{-0.19683}$
So, $x < 0$

St. II : $y = \sqrt[3]{x}$
So, $y < 0$, as $x < 0$
$\therefore xy > 0$ i.e., positive.

65. Option (b) is correct.
Explanation: Given: a, b, c are sides of ΔABC
St I : $a^2 + b^2 + c^2 = ab + bc + ca$
$\Rightarrow a = b = c$
$\therefore$ Triangle is an equilateral
St. II : $3a^2 + 3b^2 + 4c^2 = 2ab + 4bc + 4ac$

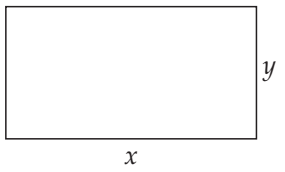
$\Rightarrow (a-b)^2 + (\sqrt{2}b-\sqrt{2}c)^2 + (\sqrt{2}a-\sqrt{2}c)^2 = 0$

This is possible only, if $a - b = 0$, $\sqrt{2}b-\sqrt{2}c=0$

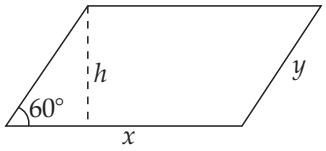
and $\sqrt{2}a-\sqrt{2}b=0$

$\Rightarrow a = b = c$
$\therefore$ Triangle is an equilateral

66. Option (c) is correct.
Explanation: Length of rectangle = x,
Breadth of rectangle = y
$\therefore$ P = xy

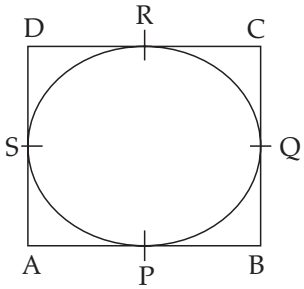


Area of ||gm, Q = $x \times h$



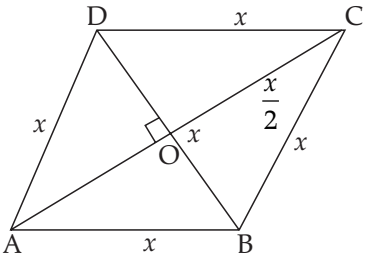
So using both statements, we can say P > Q

67. Option (b) is correct.
Explanation: $\because$ AP = AS (Tangent drawn an external point are equal)



PB = BQ, QC = RC & RD = DS
$\therefore$ AP + PB + CR + RD = AS + BQ + QC + DS
$\Rightarrow$ AB + CD = AD + BC
St. I: AB + DC = 10 = AD + BC
So, perimeter = 10 + 10 = 20 cm
St. II: AD + BC = 10 = AB + CD
So, perimeter = 20 cm

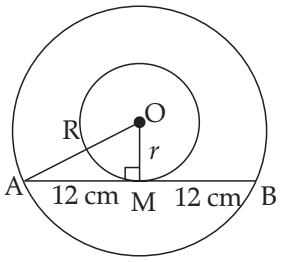
68. Option (b) is correct.
Explanation: Statement-I:
So, ΔABC is an equilateral triangle



DO = $\sqrt{x^2-\frac{x^2}{4}} = \sqrt{\frac{3x^2}{4}} = \frac{\sqrt{3}x}{2}$

So, DB = $\sqrt{3}x$
$\therefore$ ratio = $x:\sqrt{3}x = 1:\sqrt{3}$ True
Statement-II If BD = $\sqrt{3}x$
So, similarly, AC = x
$\therefore$ ratio = $x:\sqrt{3}x = 1:\sqrt{3}$

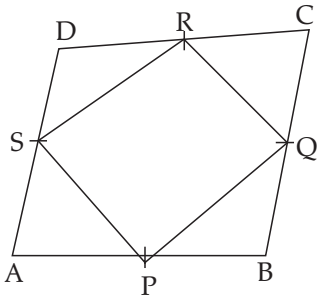
69. Option (d) is correct.
Explanation: Here, OM = r units
OA = R units



In ΔOAM,
$OA^2 = OM^2 + AM^2$
$R^2 = r^2 + 144$ (Using pythagoras theorem)
By using both statements, we cannot find exact values of r and R.

70. Option (b) is correct.
Explanation: As, P, Q, R, S are the mid-points of AB, BC, CD and AD respectively
$\therefore$ area of quad. PQRS

$= -$ area of quad. ABCD



St-I: Area of quad. PQRS $= \frac{1}{2} \times 100 = 50$ sq. units
$\therefore$ Difference = 100 – 50 = 50 sq. units
St-II: Area of quad. ABCD = 2 × 50
= 100 sq. units
$\therefore$ Difference = 100 – 50 = 50 sq. units

71. Option (d) is correct.
Explanation: Radius = 7 cm
Let the angles of sectors be $2x$, $3x$, $7x$, $5x$ and x.
$\therefore 2x + 3x + 7x + 5x + x = 360°$
$\Rightarrow 18x = 360° \Rightarrow x = 20°$
P = area of smallest sector

$$= \frac{x}{360°} \times \pi(7)^2 = \frac{20°}{360°} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{9} \text{ sq. cm}$$

Q = area of largest sector

$$= \frac{7x}{360} \times \pi(7)^2 = \frac{140°}{360°} \times \frac{22}{7} \times 49 = \frac{539}{9} \text{ sq. cm}$$

$$\therefore P + Q = \frac{77}{9} + \frac{539}{9} = \frac{616}{9} \text{ sq. cm}$$

72. Option (b) is correct.
Explanation: p = perimeter of smallest sector

$$= \left(\frac{x}{360°} \times 2 \times \frac{22}{7} \times 7\right) + 2 \times 7$$

$$= \left(\frac{20°}{360°} \times 44\right) + 14 = 16\frac{4}{9} = \frac{148}{9}$$

$$\therefore 9p = 9 \times \frac{148}{9} = 148 \text{ cm}$$

For Solution 73-74
Train A → Started at 7 p.m. → 60 km/h
Train B → Starting at 4 a.m. → 90 km/h(next day)
Distance between stations P and Q = 800 km
Distance covered by A till 4 a.m. = 9 × 60
= 540 km
$\therefore$ Remaining distance = 800 – 540 = 260 km

73. Option (c) is correct.
Explanation:

$$\therefore \text{Time} = \frac{260}{60+90} = \frac{260}{150} = \frac{26}{15} \text{ hours}$$

So, required distance $= \frac{26}{15} \times 90 = 156$ km

74. Option (b) is correct.
Explanation: So, two trains will meet $= \frac{26}{15}$ hours after 4 a.m. i.e., at 5:44 a.m.

75. Option (a) is correct.
Explanation: Length of train A = 400 m
Length of train B = 500 m

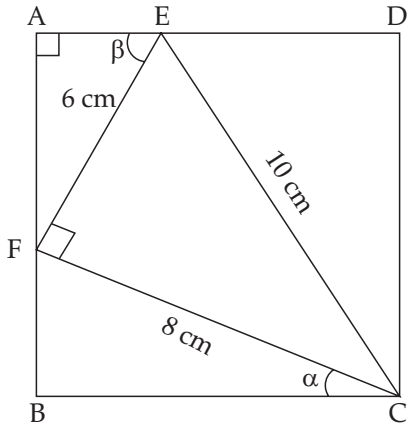
$$\text{So, required time} = \frac{400+500}{150 \times \frac{5}{18}} = \frac{900 \times 18}{750}$$

= 21.6 seconds

76. Option (c) is correct.
Explanation: In ΔCEF
$\because (CE)^2 = 100$
$(CF)^2 + (EF)^2 = 36 + 64 = 100$
$\Rightarrow \angle F = 90°$
So, $\angle BFC = 90° - \alpha$
and $\angle AFE = \alpha$
In ΔAFE
$\alpha + \beta = 90°$ ($\therefore \angle A = 90°$)
$\because \Delta AFE \sim \Delta BCF$ (By AA similarity)

$$\frac{AF}{BC} = \frac{6}{8} \Rightarrow \frac{AF}{BC} = \frac{3}{4}$$

Let AF = $3x$ & BC = $4x$
So, BF = $4x - 3x = x$



In ΔBFC, $x^2 + (4x)^2 = 8^2$

$\Rightarrow 17x^2 = 64 \Rightarrow x^2 = \dfrac{64}{17}$

$\therefore$ Area of square $= (4x)^2 = 16 \times x^2$

$= 16 \times \dfrac{64}{17} = \dfrac{1024}{17}$ sq. cm

77. Option (d) is correct.

Explanation: $\tan\alpha = \dfrac{BF}{BC} = \dfrac{x}{4x} = \dfrac{1}{4}$

$\tan\beta = \tan(90° - \alpha) = \cot\alpha = 4$

$\therefore \tan\alpha + \cot\alpha = \dfrac{1}{4} + 4 = \dfrac{17}{4}$

78. Option (a) is correct.

Explanation: In ΔCDE

$CD = 4x = 4 \times \dfrac{8}{\sqrt{17}} = \dfrac{32}{\sqrt{17}}$ cm

In ΔCDE, $(CE)^2 = (CD)^2 + (DE)^2$

$\Rightarrow DE = \sqrt{100 - \dfrac{(32)^2}{17}}$

$= \sqrt{\dfrac{1700 - 1024}{17}} = \sqrt{\dfrac{676}{17}} = \dfrac{26}{\sqrt{17}}$ cm

So, area of ΔCDE $= \dfrac{1}{2} \times CD \times DE$

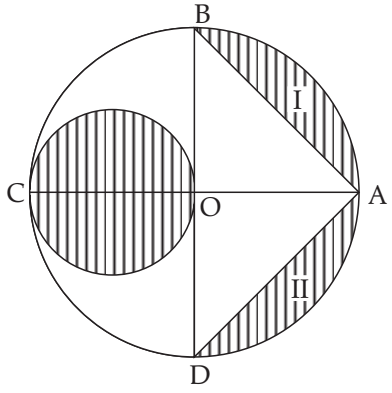
$= \dfrac{1}{2} \times \dfrac{32}{\sqrt{17}} \times \dfrac{26}{\sqrt{17}} = \dfrac{416}{17}$ sq. cm

79. Option (d) is correct.

Explanation: Here, OB = 7 cm

Area of small circle with diameter OC

$= \pi \times \left(\dfrac{7}{2}\right)^2 = \dfrac{77}{2}$ sq. cm



Area of shaded region (I and II)

= Area of semi-circle – Area of ΔABD

$= \dfrac{1}{2} \times \dfrac{22}{7} \times 7^2 - \dfrac{1}{2} \times 14 \times 7$

$= 77 - 49 = 28$

$\therefore$ Required area $= \dfrac{77}{2} + 28 = \dfrac{133}{2} = 66.5$ sq. cm

80. Option (a) is correct.

Explanation: Area of non-shaded region

= Area of big circle – Area of shaded region

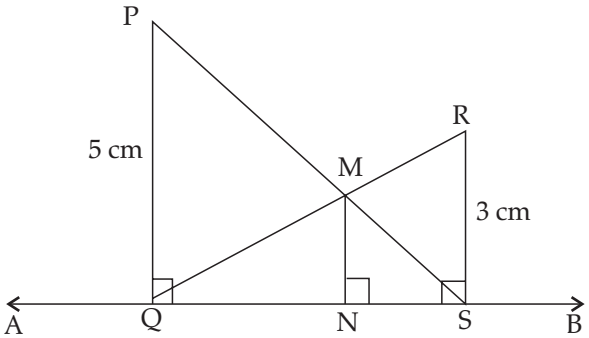
$= \dfrac{22}{7} \times 7^2 - \dfrac{133}{2} = 154 - \dfrac{133}{2} = \dfrac{175}{2}$ sq. cm

$\therefore$ Required ratio $= \dfrac{\frac{133}{2}}{\frac{175}{2}} = \dfrac{19}{25}$

81. Option (d) is correct.

Explanation: As, PQ||MN||RS

So, $\dfrac{1}{MN} = \dfrac{1}{PQ} + \dfrac{1}{RS}$



$= \dfrac{1}{3} + \dfrac{1}{5} = \dfrac{3+5}{15} = \dfrac{8}{15}$

$\Rightarrow MN = \dfrac{15}{8}$ cm

82. Option (c) is correct.
Explanation:

$$\text{Required ratio} = \frac{\text{Area of trap. PQNM}}{\text{Area of trap. NMRS}}$$

$$= \frac{\frac{1}{2}\times(PQ+MN)\times QN}{\frac{1}{2}\times(MN+RS)\times NS} \quad ...(i)$$

Since, $\Delta MNS \sim \Delta PQS$

$$\therefore \frac{MN}{PQ} = \frac{NS}{QS}$$

$$\Rightarrow \frac{QN+NS}{NS} = \frac{5}{15}\times 8 = \frac{8}{3}$$

$$\Rightarrow \frac{QN}{NS}+1 = \frac{8}{3} \Rightarrow \frac{QN}{NS} = \frac{5}{3}$$

$$\text{Using (i) Ratio} = \frac{\left(5+\frac{15}{8}\right)}{\left(\frac{15}{8}+3\right)}\times\frac{5}{3} = \frac{55}{37}\times\frac{5}{3} = \frac{275}{117}$$

83. Option (a) is correct.
Explanation: Let the total no of people be x
So, according to the question,
15% of 20% of x = 24,000

$$\Rightarrow x = \frac{24,000\times 100\times 100}{15\times 20} = 8,00,000$$

So, number of migrating people belonging to age group B = 15% of x

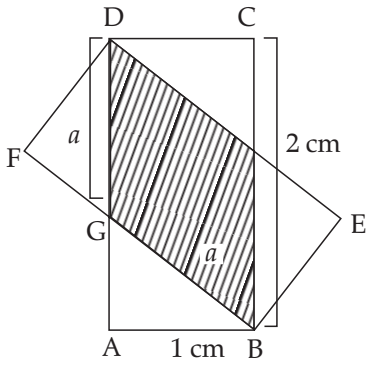
$$= \frac{15}{100}\times 8,00,000 = 1,20,000$$

84. Option (c) is correct.
Explanation: Required maximum difference

$$= (40-10)\% \text{ of } x = \frac{30}{100}\times 8,00,000 = 2,40,000$$

85. Option (b) is correct.
Explanation: Number of people coming from R having age group A
= 10% × 50% × 8,00,000 = 40,000
Number of people coming from Q having age group D = 30% × 20% × 8,00,000 = 48,000
∴ Required difference = 48,000 – 40,000 = 8,000

86. Option (b) is correct.
Explanation: Let DG = a = GB
In ΔAGB
$a^2 = (2-a)^2 + 1^2$
$\Rightarrow a^2 = 4 + a^2 - 4a + 1$



$$\Rightarrow 4a = 5 \Rightarrow a = \frac{5}{4} \text{ cm}$$

So, required area = Area of rectangle
– 2 (area of ΔAGB)

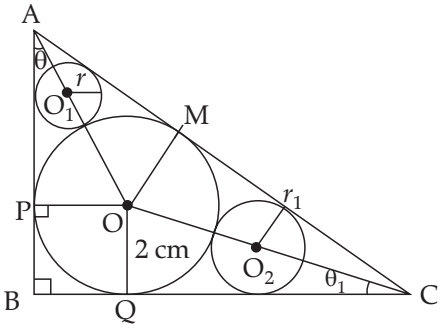
$$= (1\times 2) - 2\times\left(\frac{1}{2}\times 1\times\left(2-\frac{5}{4}\right)\right)$$

$$= \left(2-\frac{3}{4}\right) = \frac{5}{4} \text{ sq. cm}$$

87. Option (c) is correct.
Explanation: Area of non-overlapping region
= 2 [Area of rectangle] – 2 [area of overlapping region]

$$= 2\times 1\times 2 - 2\times\frac{5}{4} = 4 - \frac{5}{2} = \frac{3}{2} \text{ sq. cm}$$

88. Option (d) is correct.
Explanation: MA : MC = 2 : 3
Let AM = 2x & MC = 3x



So, AP = 2x and QC = 3x
In ΔABC
$(AC)^2 = (AB)^2 + (BC)^2$
$(5x)^2 = (2x+2)^2 + (3x+2)^2$
$\Rightarrow 25x^2 = 4x^2 + 4 + 8x + 9x^2 + 4 + 12x$
$\Rightarrow 12x^2 - 20x - 8 = 0 \Rightarrow 3x^2 - 5x - 2 = 0$
$\Rightarrow 3x^2 - 6x + x - 2 = 0$
$\Rightarrow 3x(x-2) + 1(x-2) = 0$
$\therefore x = 2$
So, AB + BC = 2x + 2 + 3x + 2 = 5x + 4
= 5(2) + 4 = 14 cm

89. Option (d) is correct.

Explanation: $\frac{r}{2}=\frac{1-\sin\theta}{1+\sin\theta}$

In ΔAOP, $\sin\theta=\frac{2}{\sqrt{4^2+2^2}}=\frac{2}{\sqrt{20}}=\frac{2}{2\sqrt{5}}=\frac{1}{\sqrt{5}}$

So, $\frac{r}{2}=\frac{1-\frac{1}{\sqrt{5}}}{1+\frac{1}{\sqrt{5}}}=\frac{\sqrt{5}-1}{\sqrt{5}+1}\times\frac{\sqrt{5}-1}{\sqrt{5}-1}$

$\Rightarrow\frac{r}{2}=\frac{5+1-2\sqrt{5}}{4}=\frac{6-2\sqrt{5}}{4}$

$\Rightarrow r=(3-\sqrt{5})$ cm

90. Option (c) is correct.

Explanation: In ΔQOC

$\sin\theta_1=\frac{2}{\sqrt{6^2+2^2}}=\frac{2}{\sqrt{40}}=\frac{2}{2\sqrt{10}}=\frac{1}{\sqrt{10}}$

$\therefore\frac{r_1}{2}=\frac{1-\frac{1}{\sqrt{10}}}{1+\frac{1}{\sqrt{10}}}=\frac{\sqrt{10}-1}{\sqrt{10}+1}\times\frac{\sqrt{10}-1}{\sqrt{10}-1}$

$\Rightarrow\frac{r_1}{2}=\frac{10+1-2\sqrt{10}}{9}=\frac{11-2\sqrt{10}}{9}$

$\Rightarrow r_1=\frac{22-4\sqrt{10}}{9}$ cm

91. Option (c) is correct.

Explanation: Since DC = 10 cm

$\therefore$ radius of semi-circle = OD = 5 cm

In semi-circle DQC,

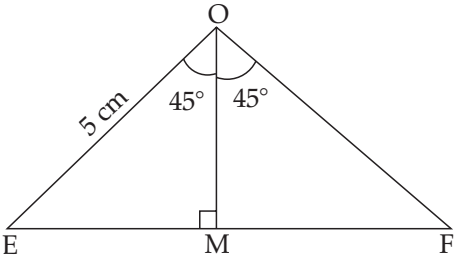
OE = 5 cm = OF (radii of circle)

So, area of $\Delta EOF=\frac{1}{2}\times OE\times OF$

$=\frac{25}{2}=12.5$ sq. cm

92. Option (a) is correct.

Explanation: In ΔOEM



$\sin45°=\frac{EM}{OE}$

$\Rightarrow\frac{1}{\sqrt{2}}=\frac{EM}{5}\Rightarrow EM=\frac{5}{\sqrt{2}}$ cm

So, EF = $5\sqrt{2}$ cm

Now, in trapezium AEFB,

EF = $5\sqrt{2}$ cm, AB = 10 cm, height = $\frac{5}{\sqrt{2}}$ cm

$\therefore$ Area of trapezium AEFB = $\frac{1}{2}\times\left(5\sqrt{2}+10\right)\times\frac{5}{\sqrt{2}}$

$=\frac{1}{2}\left\{25+\frac{50}{\sqrt{2}}\right\}$

$=\frac{1}{2}\{25+25\sqrt{2}\}=\frac{1}{2}\{25+35\}=30$ sq. cm

93. Option (b) is correct.

Explanation: Area of segment EQF = Area of sector OEQF – Area of ΔEOF

$=\frac{90°}{360°}\times3.14\times(5)^2-12.5$

= 19.625 – 12.5 = 7.125

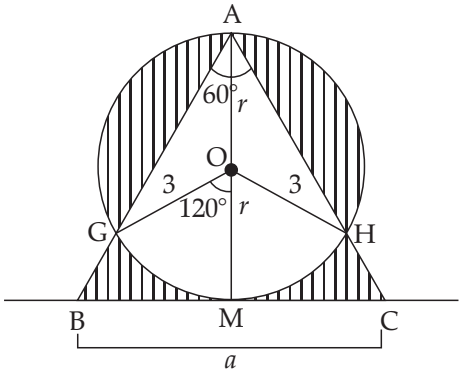
So, required shaded area = 2 × 7.125

= 14.25 sq. cm

94. Option (b) is correct.

Explanation: Here, AM = 2*r*

Let *a* be side of an equilateral triangle



So, area of $\Delta=\frac{1}{2}\times a\times2r$

$\frac{\sqrt{3}}{4}a^2=\frac{1}{2}\times a\times(2r)$

$\Rightarrow r=\frac{\sqrt{3}}{4}a$

Since, area of circle = 9π (Given)

$\Rightarrow\quad\pi\left(\frac{\sqrt{3}}{4}a\right)^2=9\pi$

$\Rightarrow\quad\frac{3}{16}a^2=9$

$\Rightarrow\quad a^2=16\times3\Rightarrow a=4\sqrt{3}$ units

95. Option (b) is correct.

Explanation: Area of $\Delta=\frac{\sqrt{3}}{4}\times(4\sqrt{3})^2=12\sqrt{3}$

Area of $\Delta AOG = \frac{1}{2}\times 3\times 3\times \sin 120°$

$= \frac{9}{2}\times\frac{\sqrt{3}}{2} = \frac{9\sqrt{3}}{4}$ sq. unit

Area of sector OGMH $= \frac{120°}{360°}\times\pi\times 9$

$= 3\pi$ sq units.

$\therefore$ Area of unshaded part

$= \left(2\times\frac{9\sqrt{3}}{4}+3\pi\right) = \left(\frac{9}{2}\sqrt{3}+3\pi\right)$ sq. unit

So, area of shaded region

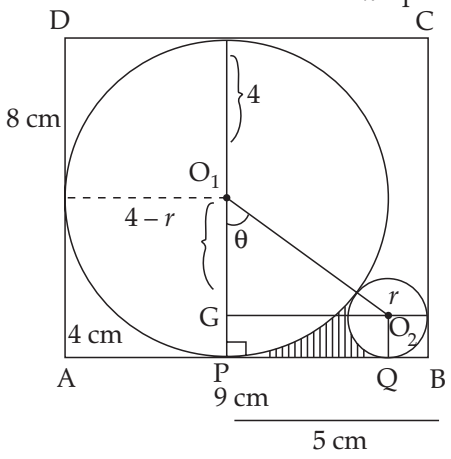
$= (9\pi+12\sqrt{3}) - 2\left[\frac{9}{2}\sqrt{3}+3\pi\right]$

$= 9\pi+12\sqrt{3}-9\sqrt{3}-6\pi \ = 3\pi+3\sqrt{3}$

96. Option (a) is correct.

Explanation: Area of big circle $= \pi(4)^2$

$= 16\pi$ sq. cm



In ΔO_1GO_2

Using Pythagoras theorem,

$(O_1O_2)^2 = (O_1G)^2 + (GO_2)^2$

$(4 + r)^2 = (4 - r)^2 + (5 - r)^2$

$\Rightarrow 16 + r^2 + 8r = 16 + r^2 - 8r + 25 + r^2 - 10r$

$\Rightarrow r^2 - 26r + 25 = 0$

$\Rightarrow (r - 25)(r - 1) = 0$

So, $r = 1$

$\therefore$ Area of small circle $= \pi(1)^2 = \pi$ sq. cm

So, required sum $= 16\pi + \pi = 17\pi$

97. Option (c) is correct.

Explanation: In ΔO_1GO_2

$OG = 4 - 1 = 3$ cm, $GO_2 = 5 - 1 = 4$ cm,

$O_1O_2 = 5$ cm

So, $\tan\theta = \frac{GO_2}{O_1G} = \frac{4}{3}$

$\therefore 45° < \theta < 60°$

98. Option (d) is correct.

Explanation: Here, $O_1P \,||\, O_2Q$

So, O_1 PQ O_2 is a trapezium

So, area of trapezium $= \frac{1}{2}\times(4+1)\times 4$

$= 10$ sq. cm

Area of sector of big circle $= \frac{\theta}{360}\times\pi(4)^2$

Area of sector of small circle $= \frac{180-\theta}{360°}\times\pi(1)^2$

$\therefore$ Required area $= 10 - \frac{16}{360}\pi\theta - \frac{1}{2}\times\pi + \frac{\theta}{360°}\pi$

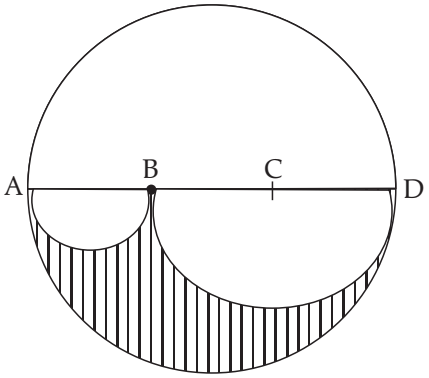
$= 10 - \frac{\pi}{2} + \frac{\theta}{360°}\pi\,(-15) = \frac{240-12\pi-\pi\theta}{24}$

99. Option (a) is correct.

Explanation: AD = 12 cm

So, AB = BC = CD = 4 cm

Area of unshaded region



$= \frac{\pi}{2}(6)^2 - \frac{\pi}{2}(2)^2 - \frac{\pi}{2}(4)^2$

$= \frac{\pi}{2}\{36-4-16\} = \frac{\pi}{2}16 = 8\pi$

Area of unshaded region $= \frac{\pi}{2}(6)^2 + \frac{\pi}{2}(4) + \frac{\pi}{2}(16)$

$= \frac{\pi}{2}\{36+4+16\} = 28\pi$

$\therefore$ Required ratio $= \frac{8\pi}{28\pi} = \frac{2}{7}$ i.e., 2 : 7

100. Option (d) is correct.

Explanation: Perimeter of the shaded region

$= \pi\,[6] + \pi\,[2] + \pi\,[4] = \pi\,[12]$ cm $= 12\pi$ cm