NDA / NA

National Defence Academy / Naval Academy

MATHEMATICS

I

QUESTION PAPER 2024

Time Allowed : 2 hrs 30 min **Total Marks : 300**

Instructions

1. *This Test Booklet contains **120** items (questions). Each item is printed in **English**. Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.*
2. *You have to mark all your responses **ONLY** on the separate Answer Sheet provided. See directions in the Answer Sheet.*
3. ***All** items carry equal marks.*
4. *Before you proceed to mark in the Answer Sheet the response to the various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions.*
5. ***Penalty for wrong answers :***
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   *(i). There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.*
   *(ii). If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   *(iii). If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.*

1. Let A and B matrices of order 3×3. If $|A| = \frac{1}{2\sqrt{2}}$ and $|B| = \frac{1}{729}$, then what is the value of $|2B[adj(3A)]|$?

(a) 27 **(b)** $\frac{27}{2\sqrt{2}}$ **(c)** $\frac{27}{2}$ **(d)** 1

2. If z is any complex number and $iz^3 + z^2 - z + i = 0$, where $i = \sqrt{-1}$, then what is the value of $(|z| + 1)^2$?

(a) 1 **(b)** 4 **(c)** 81 **(d)** 121

3. What is the sum of all four digit numbers formed by using all digits 0, 1, 4, 5 without repetition of digits?

(a) 44440 **(b)** 46460 **(c)** 46440 **(d)** 64440

4. If x, y and z are the cube roots of unity, then what is the value of $xy + yz + zx$?

(a) 0 **(b)** 1 **(c)** 2 **(d)** 3

5. A man has 7 relatives (4 women and 3 men). His wife also has 7 relatives (3 women and 4 men). In how many ways can they invite 3 women and 3 men so that 3 of them are man's relatives and 3 of them are his wife's relatives?

(a) 340 **(b)** 484 **(c)** 485 **(d)** 469

6. A triangle PQR is such that 3 points lie on the side PQ, 4 points on QR and 5 points on RP respectively. Triangles are constructed using these points as vertices. What is the number of triangles so formed?

(a) 205 **(b)** 206 **(c)** 215 **(d)** 220

7. If $\log_b a = p$, $\log_d c = 2p$ and $\log_f e = 3p$, then what is $(ace)^{\frac{1}{p}}$ equal to?

(a) bd^2f^3 **(b)** bdf **(c)** b^3d^2f **(d)** $b^2d^2f^2$

8. If $-\sqrt{2}$ and $\sqrt{3}$ are roots of the equation $a_0 + a_1x + a_2x^2 + a_3x^3 + x^4 = 0$ where a_0, a_1, a_2, a_3 are integers, then which one of the following is correct?

(a) $a_2 = a_3 = 0$ **(b)** $a_2 = 0$ and $a_3 = -5$
(c) $a_0 = 6, a_3 = 0$ **(d)** $a_1 = 0$ and $a_2 = 5$

9. Let z_1 and z_2 be two complex numbers such that $\left|\frac{z_1 + z_2}{z_1 - z_2}\right| = 1$, then what is $\text{Re}\left(\frac{z_1}{z_2}\right) + 1$ equal to?

(a) -1 **(b)** 0 **(c)** 1 **(d)** 5

10. If $26! = n8^k$, where k and n are positive integers, then what is the maximum value of k?

(a) 6 **(b)** 7 **(c)** 8 **(d)** 9

11. Consider the following statements in respect of two non-singular matrices A and B of the same order n:

1. $adj(AB) = (adj\ A)(adj\ B)$
2. $adj(AB) = adj(BA)$
3. $(AB)adj(AB) - |AB|I_n$ is a null matrix of order n

How many of the above statements are correct?

(a) None
(b) Only one statement
(c) Only two statements
(d) All three statements

12. Consider the following statements in respect of non-singular matrix A of order n:
1. $A(\text{adj } A^T) = A(\text{adj } A)^T$
2. If $A^2 = A$, then A is identity matrix of order n
3. If $A^3 = A$, then A is identity matrix of order n

Which of the statements given above are correct?
(a) 1 and 2 only **(b)** 2 and 3 only
(c) 1 and 3 only **(d)** 1, 2 and 3

13. How many four-digit natural numbers are there such that all of the digits are even?
(a) 625 **(b)** 500 **(c)** 400 **(d)** 256

14. If $\omega \neq 1$ is a cube root of unity, then what are the solutions of $(z - 100)^3 + 1000 = 0$?
(a) $10(1 - \omega), 10(10 - \omega^2), 100$
(b) $10(10 - \omega), 10(10 - \omega^2), 90$
(c) $10(1 - \omega), 10(10 - \omega^2), 1000$
(d) $(1 + \omega), (10 + \omega^2), -1$

15. What is $(1 + i)^4 + (1 - i)^4$ equal to, where $i = \sqrt{-1}$?
(a) 4 **(b)** 0 **(c)** –4 **(d)** –8

16. Consider the following statements in respect of a skew-symmetric matrix A of order 3:
1. All diagonal elements are zero.
2. The sum of all the diagonal elements of the matrix is zero.
3. A is orthogonal matrix.

Which of the statements given above are correct?
(a) 1 and 2 only **(b)** 2 and 3 only
(c) 1 and 3 only **(d)** 1, 2 and 3

17. Four digit numbers are formed by using the digits 1, 2, 3, 5 without repetition of digits. How many of them are divisible by 4?
(a) 120 **(b)** 24 **(c)** 12 **(d)** 6

18. What is the remainder when 2^{120} is divided by 7?
(a) 1 **(b)** 3 **(c)** 5 **(d)** 6

19. For what value of n is the determinant
$$\begin{vmatrix} C(9,4) & C(9,3) & C(10,n-2) \\ C(11,6) & C(11,5) & C(12,n) \\ C(m,7) & C(m,6) & C(m+1,n+1) \end{vmatrix} = 0$$
for every $m > n$?
(a) 4 **(b)** 5 **(c)** 6 **(d)** 7

20. If ABC is a triangle, then what is the value of the determinant
$$\begin{vmatrix} \cos C & \sin B & 0 \\ \tan A & 0 & \sin B \\ 0 & \tan(B+C) & \cos C \end{vmatrix} = ?$$
(a) –1 **(b)** 0 **(c)** 1 **(d)** 3

21. What is the number of different matrices, each having 4 entries that can be formed using 1, 2, 3, 4 (repetition is allowed)?
(a) 72 **(b)** 216 **(c)** 254 **(d)** 768

22. Let $A = \{x \in R: -1 < x < 1\}$. Which of the following is/are bijective functions from A to itself?
1. $f(x) = x|x|$
2. $g(x) = \cos(\pi x)$

Select the correct answer using the code given below:
(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

23. Let R be a relation on the open interval (–1, 1) and is given by
$R = \{(x, y): |x + y| < 2\}$. Then which one of the following is correct?
(a) R is reflexive but neither symmetric nor transitive
(b) R is reflexive and symmetric but not transitive
(c) R is reflexive and transitive but not symmetric
(d) R is an equivalence relation

24. For any three non-empty sets A, B, C, what is $(A \cup B) - \{(A - B) \cup (B - A) \cup (A \cap B)\}$ equal to?
(a) Null set **(b)** A
(c) B **(d)** $(A \cup B) - (A \cap B)$

25. If a, b, c are the sides of triangle ABC, then what is
$$\begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos A \\ c\sin A & \cos A & 1 \end{vmatrix}$$
equal to?
(a) Zero
(b) Area of triangle
(c) Perimeter of triangle
(d) $a^2 + b^2 + c^2$

26. If a, b, c are in AP; b, c, d are in GP; c, d, e are in HP, then which of the following is/are correct?
1. a, c and e are in GP
2. $\frac{1}{a}, \frac{1}{c}, \frac{1}{e}$ are in GP

Select the correct answer using the code given below:
(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

27. What is the number of solutions of $\log_4(x - 1) = \log_2(x - 3)$?
(a) Zero **(b)** One **(c)** Two **(d)** Three

28. For $x \ge y > 1$, let $\log_x\left(\frac{x}{y}\right)+\log_y\left(\frac{y}{x}\right) = k$, then the value of k can never be equal to

(a) -1 **(b)** $-\frac{1}{2}$ **(c)** 0 **(d)** 1

29. If $A = \begin{bmatrix} \sin 2\theta & 2\sin^2\theta - 1 & 0 \\ \cos 2\theta & 2\sin\theta\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then which of the following statements is/are correct?

1. $A^{-1} = adj\, A$

2. A is skew-symmetric matrix

3. $A^{-1} = A^T$

Select the correct answer using the code given below:

(a) 1 only **(b)** 1 and 2

(c) 1 and 3 **(d)** 2 and 3

30. What is the coefficient of x^{10} in the expansion of $(1-x^2)^{20}\left(2-x^2-\frac{1}{x^2}\right)^{-5}$?

(a) -1

(b) 1

(c) 10

(d) Coefficient of x^{10} does not exist

31. If the 4th term in the expansion of $\left(mx+\frac{1}{x}\right)^n$ is $\frac{5}{2}$, then what is the value of mn?

(a) -3 **(b)** 3 **(c)** 6 **(d)** 12

32. If a, b and c ($a > 0, c > 0$) are in GP, then consider the following in respect of the equation $ax^2 + bx + c = 0$;

1. The equation has imaginary roots.

2. The ratio of the roots of the equation is 1: ω where ω is a cube root of unity.

3. The product of roots of the equation is $\left(\frac{b^2}{a^2}\right)$.

Which of the statements given above are correct?

(a) 1 and 2 only **(b)** 2 and 3 only

(c) 1 and 3 only **(d)** 1, 2 and 3

33. If $x^2 + mx + n$ is an integer form all integral values of x, then which of the following is/are correct?

1. m must be an integer

2. n must be an integer

Select the correct answer using the code given below:

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

34. In a binomial expansion of $(x + y)^{2n+1}(x - y)^{2n+1}$, the sum of middle terms is zero. What is the value of $\left(\frac{x^2}{y^2}\right)$?

(a) 1 **(b)** 2 **(c)** 4 **(d)** 8

35. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{6, 7\}$. What is the number of onto functions from A to B?

(a) 10 **(b)** 20 **(c)** 30 **(d)** 32

36. What is $\frac{\sqrt{3}\cos 10° - \sin 10°}{\sin 25° \cos 25°}$ equal to?

(a) 1 **(b)** $\sqrt{3}$ **(c)** 2 **(d)** 4

37. What is $(\sin 9° - \cos 9°)$ equal to?

(a) $-\frac{\sqrt{5-\sqrt{5}}}{2}$ **(b)** $-\frac{\sqrt{5-\sqrt{3}}}{2}$

(c) $\frac{\sqrt{5-\sqrt{5}}}{2}$ **(d)** $\frac{\sqrt{5-\sqrt{5}}}{4}$

38. If in a triangle ABC, $\sin^3 A + \sin^3 B + \sin^3 C = 3\sin A \sin B \sin C$, then what is the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$; where a, b, c are sides of the triangle?

(a) $a + b + c$

(b) $ab + bc + ca$

(c) $(a + b)(b + c)(c + a)$

(d) 0

39. If $\cos^{-1}x = \sin^{-1}x$, then which one of the following is correct?

(a) $x = 1$ **(b)** $x = \frac{1}{2}$

(c) $x = \frac{1}{\sqrt{2}}$ **(d)** $x = \frac{1}{\sqrt{3}}$

40. What is the number of solutions of $(\sin\theta - \cos\theta)^2 = 2$ where $-\pi < \theta < \pi$?

(a) Only one **(b)** Only two

(c) Four **(d)** No solution

41. ABC is a triangle such that angle $C = 60°$, then what is $\frac{\cos A + \cos B}{\cos\left(\frac{A-B}{2}\right)}$ equal to?

(a) 2 (b) $\sqrt{2}$ (c) 1 (d) $\frac{1}{\sqrt{2}}$

42. What is $\sqrt{15+\cot^2\left(\frac{\pi}{4}-2\cot^{-1}3\right)}$ equal to?

(a) 1 (b) 7 (c) 8 (d) 16

43. What is the value of sin 10° . sin 50° + sin 50° . sin 250° + sin 250° . sin 10° equal to?

(a) $-\frac{1}{4}$ (b) $-\frac{3}{4}$

(c) $\frac{3\sin 10°}{4}$ (d) $-\frac{3\cos 10°}{4}$

44. What is $\tan^{-1}\left(\frac{a}{b}\right)-\tan^{-1}\left(\frac{a-b}{a+b}\right)$ equal to?

(a) $-\frac{\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $\tan^{-1}\left(\frac{a^2-b^2}{a^2+b^2}\right)$ (d) $\tan^{-1}\left(\frac{2ab}{a^2+b^2}\right)$

45. Under which one of the following conditions does the equation $(\cos\beta-1)x^2+(\cos\beta)x+\sin\beta=0$ in x have a real root for $\beta\in[0,\pi]$?

(a) $1-\cos\beta<0$ (b) $1-\cos\beta\le 0$

(c) $1-\cos\beta>0$ (d) $1-\cos\beta\ge 0$

46. In a triangle ABC, $AB = 16$ cm, $BC = 63$ cm and $AC = 65$ cm. What is the value of cos 2A + cos 2B + cos 2C?

(a) –1 (b) 0 (c) 1 (d) $\frac{76}{65}$

47. If $f(\theta)=\frac{1}{1+\tan\theta}$ and $\alpha+\beta=\frac{5\pi}{4}$, then what is the value of $f(\alpha)\,f(\beta)$?

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

48. If tan α and tan β are the roots of the equation $x^2-6x+8=0$, then what is the value of $\cos(2\alpha+2\beta)$?

(a) $\frac{13}{75}$ (b) $\frac{13}{85}$ (c) $\frac{17}{85}$ (d) $\frac{19}{85}$

49. What is the value of tan 65° + 2tan 45° – 2tan 40° – tan 25°?

(a) 0 (b) 1 (c) 2 (d) 4

50. Consider the following statements:

1. In a triangle ABC, if cot A . cot B . cot C > 0, then the triangle is an acute angled triangle.

2. In a triangle ABC, if tan A . tan B . tan C > 0, then the triangle is an obtuse angled triangle.

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

51. If (a, b) is the centre and c is the radius of the circle $x^2+y^2+2x+6y+1=0$, then what is the value of $a^2+b^2+c^2$?

(a) 19 (b) 18 (c) 17 (d) 11

52. If (1, –1, 2) and (2, 1, –1) are the end points of a diameter of a sphere $x^2+y^2+z^2+2ux+2vy+2wz-1=0$, then what is $u+v+w$ equal to?

(a) −2 (b) –1 (c) 1 (d) 2

53. The number of points represented by the equation $x = 5$ on the xy-plane is

(a) Zero (b) One

(c) Two (d) Infinitely many

54. If $< l, m, n >$ are the direction cosines of a normal to the plane $2x-3y+6z+4=0$, then what is the value of $49(7l^2+m^2-n^2)$?

(a) 0 (b) 1 (c) 3 (d) 71

55. A line through (1, –1, 2) with direction ratios < 3, 2, 2 > meets the plane $x+2y+3z=18$. What is the point of intersection of line and plane?

(a) (4, 4, 1) (b) (2, 4, 1)

(c) (4, 1, 4) (d) (3, 4, 7)

56. If p is the perpendicular distance from origin to the plane passing through (1, 0, 0), (0, 1, 0) and (0, 0, 1), then what is $3p^2$ equal to?

(a) 4 (b) 3 (c) 2 (d) 1

57. If the direction cosines $< l, m, n >$ of a line are connected by relation $l+2m+n=0$, $2l-2m+3n=0$, then what is the value of $l^2+m^2-n^2$?

(a) $\frac{1}{101}$ (b) $\frac{29}{101}$ (c) $\frac{41}{101}$ (d) $\frac{92}{101}$

58. If a variable line passes through the point of intersection of the lines $x+2y-1=0$ and $2x-y-1=0$ and meets the coordinate axis in A and B, then what is the locus of the mid-point of AB?

(a) $3x+y=10xy$ (b) $x+3y=10xy$

(c) $3x+y=10$ (d) $x+3y=10$

59. What is the equation to the straight line passing through the point (–sinθ, cosθ) and perpendicular to the line $x\cos\theta+y\sin\theta=9$?

(a) $x\sin\theta-y\cos\theta-1=0$

(b) $x\sin\theta-y\cos\theta+1=0$

(c) $x\sin\theta-y\cos\theta=0$

(d) $x\cos\theta-y\sin\theta+1=0$

60. Two points P and Q lie on line $y = 2x + 3$. These two points P and Q are at a distance 2 units from another point $R(1, 5)$. What are the coordinates of the points P and Q?

(a) $\left(1+\frac{2}{\sqrt{5}}, 5+\frac{4}{\sqrt{5}}\right), \left(1-\frac{2}{\sqrt{5}}, 5-\frac{4}{\sqrt{5}}\right)$

(b) $\left(3+\frac{2}{\sqrt{5}}, 5+\frac{4}{\sqrt{5}}\right), \left(-1-\frac{2}{\sqrt{5}}, 5-\frac{4}{\sqrt{5}}\right)$

(c) $\left(1-\frac{2}{\sqrt{5}}, 5+\frac{4}{\sqrt{5}}\right), \left(1+\frac{2}{\sqrt{5}}, 5-\frac{4}{\sqrt{5}}\right)$

(d) $\left(3-\frac{2}{\sqrt{5}}, 5+\frac{4}{\sqrt{5}}\right), \left(-1+\frac{2}{\sqrt{5}}, 5-\frac{4}{\sqrt{5}}\right)$

61. If two sides of a square lie on the lines $2x + y - 3 = 0$ and $4x + 2y + 5 = 0$, then what is the area of the square in square units?

(a) 6.05 **(b)** 6.15 **(c)** 6.25 **(d)** 6.35

62. ABC is a triangle with $A(3, 5)$. The mid-points of sides AB, AC are at $(-1, 2)$, $(6, 4)$ respectively. What are the coordinates of centroid of the triangle ABC?

(a) $\left(\frac{8}{3}, \frac{11}{3}\right)$ **(b)** $\left(\frac{7}{3}, \frac{7}{3}\right)$

(c) $\left(2, \frac{8}{3}\right)$ **(d)** $\left(\frac{8}{3}, 2\right)$

63. ABC is an acute angled isosceles triangle. Two equal sides AB and AC lie on the lines $7x - y - 3 = 0$ and $x + y - 5 = 0$. If θ is one of the equal angles, then what is $\cot \theta$ equal to?

(a) $\frac{1}{3}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{2}{3}$ **(d)** 2

64. In the parabola $y^2 = 8x$, the focal distance of a point P lying on it is 8 units. Which of the following statements is/are correct?

1. The coordinates of P can be $(6, 4\sqrt{3})$.
2. The perpendicular distance of P from the directrix of parabola is 8 units.

Select the correct answer using the code given below:

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

65. What is the eccentricity of the ellipse if the angle between the straight lines joining the foci to an extremity of the minor axis is 90°?

(a) $\frac{1}{3}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{\sqrt{3}}$ **(d)** $\frac{1}{\sqrt{2}}$

66. Let $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. If $\vec{a} \times (\vec{b} \times \vec{a}) = \alpha\hat{i} - \beta\hat{j} + \gamma\hat{k}$, then what is the value of $\alpha + \beta + \gamma$?

(a) 8 **(b)** 7 **(c)** 6 **(d)** 0

67. If a vector of magnitude 2 units makes an angle $\frac{\pi}{3}$ with $2\hat{i}$, $\frac{\pi}{4}$ with $3\hat{j}$ and an acute angle θ with $4\hat{k}$, then what are the components of the vector?

(a) $(1, \sqrt{2}, 1)$ **(b)** $(1, -\sqrt{2}, 1)$

(c) $(1, -\sqrt{2}, -1)$ **(d)** $(1, \sqrt{2}, -1)$

68. Consider the following in respect of moment of a force:

1. The moment of force about a point is independent of point of application of force.
2. The moment of a force about a line is a vector quantity.

Which of the statements given above is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

69. For any vector $\vec{r}$, what is $(\vec{r}.\hat{i})(\vec{r} \times \hat{i}) + (\vec{r}.\hat{j})(\vec{r} \times \hat{j}) + (\vec{r}.\hat{k})(\vec{r} \times \hat{k})$ equal to?

(a) $\vec{0}$ **(b)** $\vec{r}$ **(c)** $2\vec{r}$ **(d)** $3\vec{r}$

70. Let $\vec{a}$ and $\vec{b}$ are two vectors of magnitude 4 inclined at an angle $\frac{\pi}{3}$, then what is the angle between $\vec{a}$ and $\vec{a} - \vec{b}$?

(a) $\frac{\pi}{2}$ **(b)** $\frac{\pi}{3}$ **(c)** $\frac{\pi}{4}$ **(d)** $\frac{\pi}{6}$

71. Let $y_1(x)$ and $y_2(x)$ be two solutions of the differential equation $\frac{dy}{dx} = x$. If $y_1(0) = 0$ and $y_2(0) = 4$, then what is the number of points of intersection of the curves $y_1(x)$ and $y_2(x)$?

(a) No point

(b) One point

(c) Two points

(d) More than two points

72. The differential equation, representing the curve $y = e^x(a \cos x + b \sin x)$ where a and b are arbitrary constants, is

(a) $\frac{d^2y}{dx^2}+2y=0$

(b) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=0$

(c) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

(d) $\frac{d^2y}{dx^2}+y=0$

73. If $f(x) = ax - b$ and $g(x) = cx + d$ are such that $f[g(x)] = g[f(x)]$, then which one of the following holds?

(a) $f(d) = g(b)$ **(b)** $f(b) + g(d) = 0$
(c) $f(a) + g(c) = 2a$ **(d)** $f(d) + g(b) = 2d$

74. What is $\int_{-1}^{1}(3\sin x-\sin 3x)\cos^2 x\,dx$ equal to?

(a) $-\frac{1}{4}$ **(b)** 0 **(c)** $\frac{1}{2}$ **(d)** $\frac{1}{4}$

75. What are the order and degree respectively of the differential equation $\left\{2-\left(\frac{dy}{dx}\right)^2\right\}^{0.6}=\frac{d^2y}{dx^2}$?

(a) 2, 2 **(b)** 2, 3 **(c)** 5, 2 **(d)** 2, 5

76. If $\frac{dy}{dx} = 2e^x y^3$, $y(0) = \frac{1}{2}$ then what is $4y^2(2 - e^x)$ equal to?

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

77. Let $p = \int_a^b f(x)dx$ and $q = \int_a^b |f(x)|\,dx$. If $f(x) = e^{-x}$, then which one of the following is correct?

(a) $p = 2q$ **(b)** $p = -q$
(c) $4p = q$ **(d)** $p = q$

78. What is $\int_0^{\frac{\pi}{2}}\frac{a+\sin x}{2a+\sin x+\cos x}dx$ equal to?

(a) $\frac{\pi}{4}$ **(b)** $\frac{\pi}{2}$ **(c)** 1 **(d)** 0

79. The non-negative values of b for which the function $\frac{16x^3}{3}-4bx^2+x$ has neither maximum nor minimum in the range $x > 0$ is

(a) $0 < b < 1$ **(b)** $1 < b < 2$
(c) $b > 2$ **(d)** $0 \le b < 1$

80. Which one of the following is correct in respect of $f(x) = \frac{1}{\sqrt{|x|-x}}$ and $g(x) = \frac{1}{\sqrt{x-|x|}}$?

(a) $f(x)$ has some domain and $g(x)$ has no domain
(b) $f(x)$ has no domain and $g(x)$ has some domain
(c) $f(x)$ and $g(x)$ have the same domain
(d) $f(x)$ and $g(x)$ do not have any domain

*Consider the following data for the next **two (02)** items that follow:*

Given that $\int\frac{3\cos x+4\sin x}{2\cos x+5\sin x}dx$

$$=\frac{\alpha x}{29}+\frac{\beta}{29}\ln|2\cos x+5\sin x|+c$$

81. What is the value of α?

(a) 7 **(b)** 13 **(c)** 17 **(d)** 26

82. What is the value of β?

(a) 7 **(b)** 13 **(c)** 17 **(d)** 26

*Consider the following data for the next **two (02)** items that follow:*

Let $f(x) = \frac{x}{\ln x}$; $(x > 1)$

83. Consider the following statements:

1. $f(x)$ is increasing in the interval (e, ∞)
2. $f(x)$ is decreasing in the interval $(1, e)$
3. $9 \ln 7 > 7 \ln 9$

Which of the statements given above are correct?

(a) 1 and 2 only **(b)** 2 and 3 only
(c) 1 and 3 only **(d)** 1, 2 and 3

84. Consider the following statements:

1. $f''(e) = \frac{1}{e}$
2. $f(x)$ attains local minimum value at $x = e$
3. A local minimum value of $f(x)$ is e

Which of the statements given above are correct?

(a) 1 and 2 only **(b)** 2 and 3 only
(c) 1 and 3 only **(d)** 1, 2 and 3

*Consider the following data for the next **two (02)** items that follow:*

Let $f(x)$ and $g(x)$ be two functions such that $g(x) = x-\frac{1}{x}$ and $fog(x) = x^3-\frac{1}{x^3}$.

85. What is $g[f(x) - 3x]$ equal to?

(a) $x^3-\frac{1}{x^3}$ **(b)** $x^3+\frac{1}{x^3}$

(c) $x^2-\frac{1}{x^2}$ **(d)** $x^2+\frac{1}{x^2}$

86. What is $f''(x)$ equal to?

(a) $-\frac{2}{x^3}$ (b) $2x+\frac{2}{x^3}$

(c) $6x + 3$ (d) $6x$

*Consider the following data for the next **two (02)** items that follow:*

Let $f(x) = |x| + 1$ and $g(x) = [x] - 1$, where [.] is the greatest integer function.

Let $h(x) = \frac{f(x)}{g(x)}$

87. Consider the following statements:

1. $f(x)$ is differentiable for all $x < 0$
2. $g(x)$ is continuous at $x = 0.0001$
3. The derivative of $g(x)$ at $x = 2.5$ is 1

Which of the statements given above are correct?

(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only (d) 1, 2 and 3

88. What is $\lim_{x\to 0-} h(x)+\lim_{x\to 0+} h(x)$ equal to?

(a) $-\frac{3}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

*Consider the following data for the next **two (02)** items that follow:*

Let $\phi(a) = \int_a^{a+100\pi} |\sin x|\, dx$

89. What is $\phi(a)$ equal to?

(a) 0 (b) a (c) $100a$ (d) 200

90. Which is $\varphi'(a)$ equal to?

(a) 0 (b) π (c) 100 (d) 200

*Consider the following for the next **two (02)** items that follow:*

A differentiable function $f(x)$ has a local maximum at $x = 0$. Let $y = 2f(x) + ax - b$.

91. Which of the following is/are correct?

1. $f'(0) = 0$
2. $f''(0) < 0$

Select the correct answer using the code given below:

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

92. The function y has a relative maxima at $x = 0$ for

(a) $a > 0, b = 0$ (b) for all b and $a = 0$

(c) for all $b > 0$ only (d) for all a and $b = 0$

*Consider the following for the next **two (02)** items that follow:*

Let $f(x) = |x - 1|$, $g(x) = [x]$ and $h(x) = f(x)g(x)$ where [.] is greatest integer function.

93. What is $\int_{-1}^{0} h(x)dx$ equal to?

(a) $-\frac{3}{2}$ (b) -1 (c) 0 (d) $\frac{1}{2}$

94. What is $\int_{0}^{2} h(x)dx$ equal to?

(a) $-\frac{3}{2}$ (b) -1 (c) 0 (d) $\frac{1}{2}$

*Consider the following for the next **two (02)** items that follow:*

Let $\int \frac{dx}{\sqrt{x+1}-\sqrt{x-1}} = \alpha(x+1)^{\frac{3}{2}} + \beta(x-1)^{\frac{3}{2}} + c$

95. What is the value of α?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$

96. What is the value of β?

(a) $-\frac{2}{3}$ (b) $-\frac{1}{3}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

*Consider the following for the next **two (02)** items that follow:*

The circle $x^2 + y^2 - 2x = 0$ is partitioned by line $y = x$ in two segments. Let A_1, A_2 be the areas of major and minor segments respectively.

97. What is the value of A_1?

(a) $\frac{\pi-2}{4}$ (b) $\frac{\pi+2}{4}$

(c) $\frac{3\pi-2}{4}$ (d) $\frac{3\pi+2}{4}$

98. What is the value of $\frac{2(A_1+A_2)}{A_1-3A_2}$?

(a) π (b) 1 (c) -1 (d) $-\pi$

*Consider the following for the next **two (02)** items that follow:*

Let $3f(x) + f\left(\frac{1}{x}\right) = \frac{1}{x}+1$

99. What is $f(x)$ equal to?

(a) $\frac{1}{8x}-\frac{x}{8}+\frac{1}{4}$ (b) $\frac{3}{8x}-\frac{x}{8}+\frac{3}{4}$

(c) $\frac{3}{8x}+\frac{x}{8}+\frac{1}{4}$ (d) $\frac{3}{8x}-\frac{x}{8}+\frac{1}{4}$

100. What is $8\int_1^2 f(x)dx$ equal to?

(a) $\ln(8\sqrt{e})$ **(b)** $\ln(4\sqrt{e})$

(c) $\ln 2$ **(d)** $\ln 2 - 1$

101. A bag contains 5 black and 4 white balls. A man selects two balls at random. What is the probability that both of these are of the same colour?

(a) $\frac{1}{6}$ **(b)** $\frac{5}{108}$ **(c)** $\frac{4}{9}$ **(d)** $\frac{5}{18}$

102. If a random variable (x) follows binomial distribution with mean 5 and variance 4 and $5^{23}P(X = 3) = \lambda 4^{\lambda}$, then what is the value of λ?

(a) 3 **(b)** 5 **(c)** 23 **(d)** 25

103. From data (–4, 1), (–1, 2), (2, 7) and (3, 1), the regression line of y on x is obtained as $y = a + bx$, then what is the value of $2a + 15b$?

(a) 6 **(b)** 11 **(c)** 17 **(d)** 21

104. Let $x + 2y + 1 = 0$ and $2x + 3y + 4 = 0$ are two lines of regression computed from some bivariate data. If θ is the acute angle between them, then what is the value of 488 tan 3θ?

(a) 191 **(b)** 161 **(c)** 131 **(d)** 121

105. If two random variables X and Y are connected by relation $\frac{2X-3Y}{5X+4Y} = 4$ and X follows Binomial distribution with parameters $n = 10$ and $p = \frac{1}{2}$, then what is the variance of Y?

a) $\frac{810}{361}$ **(b)** $\frac{9}{19}$ **(c)** $\frac{21}{361}$ **(d)** $\frac{121}{361}$

106. If a, b, c are in HP, then what is $\frac{1}{b-a}+\frac{1}{b-c}$ equal to?

1. $\frac{2}{b}$ **2.** $\frac{1}{a}+\frac{1}{c}$

3. $\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

Select the correct answer using the code given below:

(a) 1 only **(b)** 1 and 2 only

(c) 3 only **(d)** 1, 2 and 3

107. An edible oil is sold at the rates 150, 200, 250, 300 rupees per litre in four consecutive years. Assuming that an equal amount of money is spent on oil by a family in every year during these years, what is the average price of oil in rupees (approximately) per litre?

(a) 210 **(b)** 220 **(c)** 225 **(d)** 240

108. If the letters of the word "TIRUPATI" are written down at random, then what is the probability that both Ts are always consecutive?

(a) $\frac{1}{2}$ **(b)** $\frac{1}{4}$ **(c)** $\frac{1}{7}$ **(d)** $\frac{1}{14}$

109. Let $m = 77^n$. The index n is given a positive integral value at random. What is the probability that the value of m will have 1 in the units place?

(a) $\frac{1}{2}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{1}{n}$

110. Three different numbers are selected at random from the first 15 natural numbers. What is the probability that the product of two of the numbers is equal to third number?

(a) $\frac{1}{91}$ **(b)** $\frac{2}{455}$ **(c)** $\frac{1}{65}$ **(d)** $\frac{6}{455}$

*Consider the following for the next **two (02)** items that follow:*

Let A and B be two events such that $P(A \cup B) \geq 0.75$ and $0.125 \leq P(A \cap B) \leq 0.375$.

111. What is the minimum value of $P(A) + P(B)$?

(a) 0.625 **(b)** 0.750 **(c)** 0.825 **(d)** 0.875

112. What is the maximum value of $P(A) + P(B)$?

(a) 0.75 **(b)** 1.125 **(c)** 1.375 **(d)** 1.625

*Consider the following for the next **two (02)** items that follow:*

A, B and C are three events such that $P(A) = 0.6, P(B) = 0.4, P(C) = 0.5, P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$ and $P(A \cap B \cap C) = 0.2$ and $P(A \cup B \cup C) \geq 0.85$.

113. What is the minimum value of $P(B \cap C)$?

(a) 0.1 **(b)** 0.2 **(c)** 0.35 **(d)** 0.45

114. What is the maximum value of $P(B \cap C)$?

(a) 0.1 **(b)** 0.2 **(c)** 0.35 **(d)** 0.45

*Consider the following for the next **two (02)** items that follow:*

An unbiased coin is tossed n times. The probability of getting at least one tail is p and the probability of at least two tails is q and $p - q = \frac{5}{32}$.

115. What is the value of n?

(a) 4 **(b)** 5 **(c)** 6 **(d)** 7

116. What is the value of $p + q$?

(a) $\frac{57}{32}$ **(b)** $\frac{53}{32}$ **(c)** $\frac{51}{32}$ **(d)** 1

*Consider the following for the next **two (02)** items that follow:*

x_i	1	2	3	...	n
y_i	1	2^{-1}	2^{-2}	...	$2^{-(n-1)}$

117. What is $\sum_i^n x_i f_i$ equal to?

(a) $\frac{2^{n+1}-n+2}{2^{n-1}}$ **(b)** $\frac{2^{n+1}-n-2}{2^{n-1}}$

(c) $\frac{2^{n+1}+n+2}{2^{n-1}}$ **(d)** $\frac{2^{n+1}-n-2}{2^{n}}$

118. What is the mean of the distribution?

(a) $\frac{2^{n+1}-n+2}{2^{n}-1}$ **(b)** $\frac{2^{n+1}-n-2}{2^{n-1}}$

(c) $\frac{2^{n+1}-n-2}{2^{n}-1}$ **(d)** $\frac{2^{n+1}-n+2}{2^{n}}$

*Consider the following for the next **two (02)** items that follow:*

The marks obtained by 10 students in a Statistics test are 24, 47, 18, 32, 19, 15, 21, 35, 50 and 41.

119. What is the mean deviation of the largest five observations?

(a) 4.8 **(b)** 5.5 **(c)** 6 **(d)** 7.5

120. What is the variance of the largest five observations?

(a) 14.6 **(b)** 21.8 **(c)** 25.2 **(d)** 46.8

Answer Key

Q. No	Answer Key	Topic's Name	Chapter's Name
1	d	Adjoint Matrix	Matrices and Determinants
2	b	Power of Complex Number	Complex Numbers
3	d	Sum of Number	Permutations and Combinations
4	a	Cube roots of Units	Complex Numbers
5	c	Combination	Permutations and Combinations
6	a	Combination	Permutations and Combinations
7	a	Logarithm	Relations and Functions
8	c	Roots of equation	Theory of equations
9	c	Modulus	Complex Numbers
10	b	Power of Prime Number	Permutations and Combinations
11	b	Adjoint Matrix	Matrices and Determinants
12	d	Identity matrix	Matrices and Determinants
13	b	Permutation	Permutations and Combinations
14	b	Cube roots of Units	Complex Numbers
15	d	Power of complex number	Complex Numbers
16	a	Skew Symmetric Matrix	Matrices and Determinants
17	d	Permutation	Permutation and Combination
18	a	Remainder	Binomial
19	c	Value of Determinant	Matrices and Determinants
20	b	Value of Determinant	Matrices and Determinants
21	d	Number of Matrices	Matrices and Determinants
22	a	One-One & Onto function	Relations and Functions
23	d	Equivalence Relation	Relations and Functions
24	a	Operation on Sets	Sets
25	a	Values of Determinant	Matrices and Determinants
26	a	A.P. and G.P.	Sequence and Series
27	b	Logarithmic Function	Relations and Functions
28	d	Logarithmic Function	Relations and Functions
29	c	Inverse Matrix	Matrices and Determinants
30	a	General Term	Binomial
31	b	General Term	Binomial
32	d	Solution	Quadratic Equation
33	c	Solution	Quadratic Equation
34	a	Middle Term	Binomial
35	c	Onto Functions	Relations and Functions
36	d	Values of Trigonometric ratios	Trigonometry
37	c	Values of Trigonometric ratios	Trigonometry
38	d	Values of Determinant	Matrices and Determinants
39	c	Values of Inverse Trigonometric	Inverse Trigonometry
40	b	Solution	Trigonometry
41	c	Solution	Trigonometry

Q. No	Answer Key	Topic's Name	Chapter's Name
42	c	Solution	Inverse Trigonometry
43	b	Values	Trigonometry
44	b	Simplification	Inverse Trigonometry
45	b	Simplification	Trigonometry
46	c	Simplification	Trigonometry
47	b	Sum of Angles	Trigonometry
48	b	Sum of Angles	Trigonometry
49	c	Values	Trigonometry
50	a	Triangle	Trigonometry
51	a	Circle	Conic-Section
52	a	Sphere	3-Dimensional Geometry
53	d	Equation	Straight line
54	b	Direction cosines	3-Dimensional Geometry
55	c	Plane	3-Dimensional Geometry
56	d	Plane	3-Dimensional Geometry
57	b	Direction Cosines	3-Dimensional Geometry
58	b	Family of Lines	Straight Line
59	b	Equation of Lines	Straight Line
60	a	Equation of Lines	Straight Line
61	a	Intersection of Lines	Straight Line
62	b	Centroid	Co-ordinate Geometry
63	b	Angle between lines	Straight Line
64	c	Parabola	Conic-Section
65	d	Ellipse	Conic-Section
66	d	Cross Triple Product	Vectors
67	a	Angle between two vector	Vectors
68	b	Moment of Force	Vectors
69	a	Cross Product	Vectors
70	b	Angle between vectors	Vectors
71	a	Variable Separable	Differential Equation
72	c	Formation	Differential Equation
73	d	Composite Function	Relation and Function
74	b	Definite Integral	Integration
75	d	Order and Degree	Differential Equation
76	a	Variable Separable	Differential Equation
77	d	Definite Integral	Integration
78	a	Definite Integral	Integration
79	d	Maxima and Minima	Application of Derivative
80	a	Domain of functions	Relations and Functions
81	d	Indefinite Integral	Integration
82	a	Indefinite Integral	Integration
83	d	Increasing and Decreasing	Application and Derivative
84	d	Maxima and Minima	Application and Derivative
85	a	Composite Function	Relation and Function

Q. No	Answer Key	Topic's Name	Chapter's Name
86	**d**	2nd Order Derivative	Differentiation
87	**a**	Differentiability	Differentiation
88	**a**	Limit	Limits and Derivatives
89	**d**	Definite Integral	Integration
90	**a**	Derivative	Differentiation
91	**c**	2nd Order Derivative	Differentiation
92	**b**	Maxima and Minima	Application of Derivative
93	**a**	Definite Integration	Integration
94	**d**	Definite Integration	Integration
95	**a**	Indefinite Integral	Integration
96	**c**	Indefinite Integral	Integration
97	**d**	Area bounded region	Application of Integration
98	**a**	Area bounded region	Application of Integration
99	**d**	Value of a function	Functions
100	**a**	Definite Integration	Integration
101	**c**	Basic Probability	Probability
102	**c**	Binomial Distribution	Probability
103	**b**	Regression Line	Statistics
104	**a**	Angle between lines	Straight Line
105	**a**	Variance	Statistics
106	**b**	H.P.	Sequence and Series
107	**c**	Average	Statistics
108	**b**	Basic probability	Probability
109	**c**	Basic probability	Probability
110	**d**	Basic probability	Probability
111	**d**	Properties of probability	Probability
112	**b**	Properties of probability	Probability
113	**b**	Properties of probability	Probability
114	**c**	Properties of probability	Probability
115	**b**	Binomial Distribution	Probability
116	**a**	Binomial Distribution	Probability
117	**b**	Mean	Statistics
118	**c**	Mean	Statistics
119	**c**	Mean Deviation	Statistics
120	**d**	Variance	Statistics

ANSWERS WITH EXPLANATION

1. Option (d) is correct.

$$\begin{aligned}|2B[\text{adj}(3A)]| &= |2B||\text{adj}(3A)|\\ &= 2^3|B||3A|^{3-1}\\ &= 8|B|(3^3|A|)^2\\ &= 8|B|3^6|A|^2\\ &= 8.\frac{1}{729}\times 3^6\left(\frac{1}{2\sqrt{2}}\right)^2\\ &= 8\times\frac{1}{8}\\ &= 1\end{aligned}$$

2. Option (b) is correct.

Given that $iz^3 + z^2 - z + i = 0$

Put $z = i$

$$i^4 + i^2 - i + i = 0$$

$\Rightarrow$ $1 - 1 - i + i = 0$

Satisfy them

$$\begin{aligned}(|z| + 1)^2 &= (|i| + 1)^2\\ &= 2^2 = 4\end{aligned}$$

3. Option (d) is correct.

Total four digit numbers start with 1.

$= 3 \times 2 \times 1 = 6$

$\therefore$ Repetition of digit $= \frac{6}{3} = 2$

Sum of digits in each place $= (0 + 4 + 5) \times 2$
$= 18$

$\therefore$ Sum of all four digit numbers start with 1
$= 1 \times 6 \times 1000 + 18 \times 100 + 18 \times 10 + 18$
$= 7998$

Total four digit numbers start with 4
$= 3 \times 2 \times 1 = 6$

Repetition of digit $= \frac{6}{3} = 2$

Sum of digits in each place $= (0 + 1 + 5) \times 2$
$= 12$

$\therefore$ Sum of all four digit numbers start with 4
$= 4 \times 6 \times 1000 + 12 \times 100 + 12 \times 10 + 12$
$= 25332$

Total four digit numbers start with 5
$= 3 \times 2 \times 1 = 6$

Repetition of digit $= \frac{6}{3} = 2$

Sum of digits in each place $= (0 + 1 + 4) \times 2$
$= 10$

Total four digit numbers start with 5
$= 6 \times 5 \times 1000 + 10 \times 100 + 10 \times 10 + 10$
$= 31110$

Total sum $= 7998 + 25332 + 31110$
$= 64440$

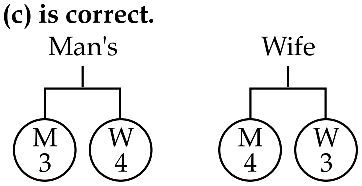
4. Option (a) is correct.

Given that x, y and z are cube roots of unity

$\therefore$ Let $x = 1, y = \omega$ and $z = \omega^2$

Now, $xy + yz + zx = \omega + \omega^3 + \omega^2$
$= \omega + 1 + \omega^2$ $(\because \omega^3 = 1)$
$= 0$

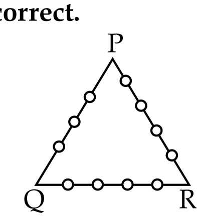
5. Option (c) is correct.



Case-1 0 3 3 0 $\rightarrow {}^3C_0 \times {}^4C_3 \times {}^4C_3 \times {}^3C_0 = 16$
Case-2 3 0 0 3 $\rightarrow {}^3C_3 \times {}^4C_0 \times {}^4C_0 \times {}^3C_3 = 1$
Case-3 1 2 2 1 $\rightarrow {}^3C_1 \times {}^4C_2 \times {}^4C_2 \times {}^3C_1 = 324$
Case-4 2 1 1 2 $\rightarrow {}^3C_2 \times {}^4C_1 \times {}^4C_1 \times {}^3C_2 = 144$

Total number of ways $= 485$

6. Option (a) is correct.



Number of triangles
$= {}^{12}C_3 - ({}^3C_3 + {}^4C_3 + {}^5C_3)$
$= 220 - (1 + 4 + 10)$
$= 205$

7. Option (a) is correct.

$$\log_b a = p$$
$$\Rightarrow a = b^p$$
$$\log_d c = 2p$$
$$\Rightarrow c = d^{2p}$$
$$\log_f e = 3p$$
$$\Rightarrow e = f^{3p}$$

Now, $(ace)^{\frac{1}{p}} = (b^p d^{2p} f^{3p})^{\frac{1}{p}}$

$= (bd^2 f^3)^{\frac{p}{p}}$

$= bd^2f^3$

8. Option (c) is correct.

Given that $-\sqrt{2}$ and $\sqrt{3}$ are roots of the given equation. Therefore, $\sqrt{2}$ and $-\sqrt{3}$ are also roots of the given equation.

$\therefore x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$

$= (x+\sqrt{2})(x-\sqrt{2})(x-\sqrt{3})(x+\sqrt{3})$

$= (x^2 - 2)(x^2 - 3)$

$= x^4 - 5x^2 + 6$

$\therefore a_3 = 0, a_2 = -5, a_1 = 0$ and $a_0 = 6$

9. Option (c) is correct.

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

Now, $\left|\frac{z_1 + z_2}{z_1 - z_2}\right| = 1$

$\Rightarrow |z_1 + z_2| = |z_1 - z_2|$

$\Rightarrow |(x_1 + x_2) + i(y_1 + y_2)|$

$= |(x_1 - x_2) + i(y_1 - y_2)|$

$\Rightarrow (x_1 + x_2)^2 + (y_1 + y_2)^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$\Rightarrow x_1^2 + x_2^2 + 2x_1x_2 + y_1^2 + y_2^2 + 2y_1y_2$

$= x_1^2 + x_2^2 - 2x_1x_2 + y_1^2 + y_2^2 - 2y_1y_2$

$\Rightarrow x_1x_2 + y_1y_2 = 0$ ...(i)

$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} \times \frac{x_2 - iy_2}{x_2 - iy_2}$

$= \frac{(x_1x_2 + y_1y_2) + i(y_1x_2 - x_1y_2)}{x_2^2 + y_2^2}$

$= 0 + \frac{i(y_1x_2 - x_1y_2)}{x_2^2 + y_2^2}$

$\therefore \mathrm{Re}\left(\frac{z_1}{z_2}\right) = 0$

Now, $\mathrm{Re}\left(\frac{z_1}{z_2}\right) + 1 = 1$

10. Option (b) is correct.

$\because 26! = n8^k = n2^{3k}$

Maximum power of 2

$= \left[\frac{26}{2}\right] + \left[\frac{26}{4}\right] + \left[\frac{26}{8}\right] + \left[\frac{26}{16}\right]$

$= 13 + 6 + 3 + 1 = 23$

But power of 2 is $3k$

$\therefore$ Maximum value of $3k = 21$

$\Rightarrow k = 7$

11. Option (b) is correct.

(1) $\because adj(AB) = (adj\ B)(adj\ A)$

$\therefore$ Statement 1 is wrong.

(2) $\because AB \neq BA$

$\therefore adj(AB) \neq adj(BA)$

$\therefore$ Statement 2 is wrong.

(3) $(AB)\ adj\ (AB) - |AB|\mathrm{I}_n$

$\Rightarrow |AB|\ \mathrm{I}_n - |AB|\ \mathrm{I}_n$ $[\because A.\ adj\ A = |A|\mathrm{I}]$

= Null matrix

$\therefore$ Statement 3 is correct.

12. Option (d) is correct.

By properties statement 1 is correct.

We know that If $A^n = A$ then A is identity matrix.

$\therefore$ Statements 2 and 3 are correct.

So, all statements are correct.

13. Option (b) is correct.

Even digit = 0, 2, 4, 6, 8.

Choice → 4 5 5 5

$\therefore$ Total required number $= 4 \times 5 \times 5 \times 5 = 500$

14. Option (b) is correct.

$(z - 100)^3 + 1000 = 0$

$\Rightarrow (z - 100)^3 = (-10)^3$

$\therefore z - 100 = -10\ (w, w^2, 1)$

$\therefore z - 100 = -10$

$\Rightarrow z = 90$

$z - 100 = -10w$

$\Rightarrow z = 100 - 10w$

$z - 100 = -10w^2$

$\Rightarrow z = 100 - 10w^2$

15. Option (d) is correct.

$(1 + i)^4 + (1 - i)^4 = [(1 + i)^2]^2 + [(1 - i)^2]^2$

$= (1 - 1 + 2i)^2 + (1 - 1 - 2i)^2$

$= (2i)^2 + (-2i)^2$

$= 4i^2 + 4i^2$

$= -4 - 4 = -8$

16. Option (a) is correct.

We know that all diagonal elements of skew symmetric matrix is zero then their sum is also zero.

So, statements 1 and 2 are correct.

We know that If $AA^T = \mathrm{I}$ then

A is called orthogonal matrix.

But $AA^T = A(-A) = -A^2$ $[\because A^T = -A]$

So, Statement 3 is wrong.

17. Option (d) is correct.

We know that if last two digit of a number divisible by 4 then number divisible by 4.

So, number is divisible by 4

$$= 1 \times 2 \times 3 \times 1$$
$$= 1 \times 2 \times 3 \times 1 = 6$$

18. Option (a) is correct.

$$2^{120} = (2^3)^{40} = 8^{40} = (1 + 7)^{40}$$
$$= 1 + {}^{40}C_1\, 7 + {}^{40}C_2\, 7^2 + \ldots + {}^{40}C_{40}\, 7^{40}$$
$$= 1 + 7[{}^{40}C_1 + {}^{40}C_2\, 7 + \ldots + {}^{40}C_{40}\, 7^{39}]$$

$\therefore$ Remainder = 1

19. Option (c) is correct.

$$\begin{vmatrix} C(9,4) & C(9,3) & C(10, n-2) \\ C(11,6) & C(11,5) & C(12, n) \\ C(m,7) & C(m,6) & C(m+1, n+1) \end{vmatrix} = 0$$

Applying $C_3 \to C_1 + C_2 - C_3$

$$\begin{vmatrix} C(9,4) & C(9,3) & C(10,4) - C(10, n-2) \\ C(11,6) & C(11,5) & C(12,6) - (12, n) \\ C(m,7) & C(m,6) & C(m+1,7) - C(m+1, n+1) \end{vmatrix}$$

$$= 0$$

Since determinant value is zero.

$\therefore \quad C_3 = 0$

So, $\quad n - 2 = 4$

$\Rightarrow \quad n = 6$

20. Option (b) is correct.

$$\begin{vmatrix} \cos C & \sin B & 0 \\ \tan A & 0 & \sin B \\ 0 & \tan(B+C) & \cos C \end{vmatrix}$$

$$= \cos C[-\sin B.\tan(B + C)] - \tan A(\sin B \cos C)$$

$$= -\sin B.\cos C\left[\frac{\sin(B+C)}{\cos(B+C)} + \frac{\sin A}{\cos A}\right]$$

$$= -\sin B \cos C\left[\frac{\sin(B+C).\cos A + \sin A \cos(B+C)}{\cos A.\cos(B+C)}\right]$$

$$= -\sin B \cos C \frac{\sin(A+B+C)}{\cos A.\cos(B+C)}$$

$$= \frac{-\sin B \cos C \sin(\pi)}{\cos A.\cos(B+C)}$$

$$= 0$$

21. Option (d) is correct.

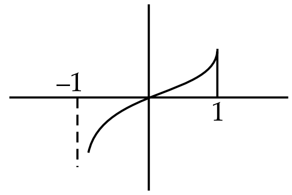
Possible order of 4 entries matrices

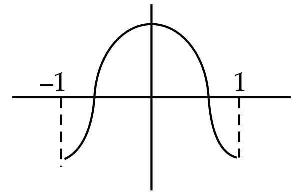
1×4, 2×2, 4×1

$\therefore$ Total number of matrices $= 4 \times 4 \times 4 \times 4 \times 3$

$$= 768$$

22. Option (a) is correct.





$f(x) = x|x|$ is one-one and onto

$g(x) = \cos(\pi x)$ is not one-one

23. Option (d) is correct.

$\because \quad xRy \Rightarrow |x + y| < 2$ in $(-1, 1)$

For reflexive

$$|x + x| < 2 \Rightarrow |x| < 1 \text{ true}$$

$\therefore$ R is reflexive

For symmetric

Let $\quad xRy \Rightarrow |x + y| < 2$

$\Rightarrow \quad |y + x| < 2 \Rightarrow yRx$

So, R is symmetric

For transitive

$\because -1 < x < 1$

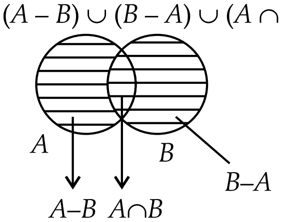
Let $\quad |x + y| < 2$ and $|y + z| < 2$

then $\quad |x + z| < 2$

So R is transitive.

24. Option (a) is correct.

$(A \cup B) - \{(A - B) \cup (B - A) \cup (A \cap B)\}$



$\therefore (A - B) \cup (B - A) \cup (A \cap B) = A \cup B$

$\therefore (A \cup B) - (A \cup B) = \varphi$ (Null set)

25. Option (a) is correct.

$$\begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos A \\ c\sin A & \cos A & 1 \end{vmatrix}$$

$= a^2(1 - \cos^2 A) - b \sin A(b \sin A - c \cos A \sin A)$
$+ \ c \sin A(b \sin A.\cos A - c \sin A)$

$= a^2 \sin^2 A - b^2 \sin^2 A + bc \sin^2 A.\cos A + bc \sin^2 A.\cos A - c^2 \sin^2 A$

$= a^2 \sin^2 A - b^2 \sin^2 A - c^2 \sin^2 A + 2bc \sin^2 A.\cos A$

$= \sin^2 A(a^2 - b^2 - c^2 + 2bc \cos A)$

$= \sin^2 A \times 0 = 0$

26. Option (a) is correct.

$\because a, b, c$ are in AP

$\Rightarrow \quad 2b = a + c \quad$ ...(i)

b, c, d are in GP

$\Rightarrow \quad c^2 = bd \quad$ ...(ii)

c, d, e are in HP

$$\Rightarrow \quad \frac{2}{d} = \frac{1}{c}+\frac{1}{e} \qquad \text{...(iii)}$$

from (i) and (ii)

$$2c^2 = (a+c)d$$

$$\Rightarrow \quad \frac{2}{d} = \frac{a+c}{c^2}$$

$$\Rightarrow \quad \frac{1}{c}+\frac{1}{e} = \frac{a}{c^2}+\frac{1}{c} \qquad \text{from (iii)}$$

$$\Rightarrow \quad \frac{1}{e} = \frac{a}{c^2}$$

$$\Rightarrow \quad c^2 = ae$$

So, a, c, e are in GP.

27. Option (b) is correct.

$$\log_4(x-1) = \log_2(x-3)$$

$$\Rightarrow \quad \frac{1}{2}\log_2(x-1) = \log_2(x-3)$$

$$\Rightarrow \quad \log_2(x-1) = \log_2(x-3)^2$$

$$\Rightarrow \quad x-1 = x^2-6x+9$$

$$\Rightarrow \quad x^2-7x+10 = 0$$

$$\Rightarrow x^2-5x-2x+10 = 0$$

$$\Rightarrow \quad (x-5)(x-2) = 0$$

$$x = 2, 5 \text{ But } x-3>0,\ x>3$$

$$\therefore \quad x = 5 \text{ only solution.}$$

28. Option (d) is correct.

$$\log_x\left(\frac{x}{y}\right)+\log_y\left(\frac{y}{x}\right) = k$$

$$\Rightarrow \log_x x - \log_x y + \log_y y - \log_y x = k$$

$$\Rightarrow \quad 1-\log_x y+1-\frac{1}{\log_x y} = k$$

Let $\quad \log_x y = t$

$$\therefore \quad 2-t-\frac{1}{t} = k$$

$$\Rightarrow \quad 2t-t^2-1 = kt$$

$$\Rightarrow \quad t^2+(k-2)^t+1 = 0$$

For solution $\quad D = b^2-4ac \geq 0$

$(k-2)^2-4 \geq 0 \quad \Rightarrow \quad (k-2)^2 \geq 4$

$k-2 \geq 2 \qquad\qquad k-2 \leq -2$

$k \geq 4 \quad$ or $\quad k \leq 0$

$\therefore k \neq 1$

29. Option (c) is correct.

$$|A| = \begin{vmatrix} \sin 2\theta & -\cos 2\theta & 0 \\ \cos 2\theta & \sin 2\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

We know that if $|A| = \pm 1$ then A is orthogonal matrix.

So, $\quad A^{-1} = A^T$ and $A^{-1} = \text{adj } A$

30. Option (a) is correct.

$$(1-x^2)^{20}\left[-\left(\frac{1}{x^2}+x^2-2\right)\right]^{-5}$$

$$= -(1-x^2)^{20}\left(x-\frac{1}{x}\right)^{-10}$$

$$= -(1-x^2)^{20}(x^2-1)^{-10}x^{10}$$

$$= -(1-x^2)^{10}.x^{10}$$

$$T_{r+1} = -{}^{10}C_r(1)^{10-r}(-x^2)^{10-r}.x^{10}$$

$$= (-1)^{11-r}.{}^{10}C_r\, x^{30-2r}$$

$$\because \quad 30-2r = 10$$

$$\Rightarrow \quad r = 10$$

$\therefore$ Coefficient of $x^{10} = (-1)^1\, {}^{10}C_{10}$

$$= -1$$

31. Option (b) is correct.

$$T_4 = T_{3+1}$$

$$= {}^nC_3(mx)^{n-3}\left(\frac{1}{x}\right)^3 = \frac{5}{2}$$

$$\frac{5}{2}x^0 = {}^nC_3(m)^{n-3}\, x^{n-6}$$

On comparing the power of x

$$\therefore \quad n-6 = 0$$

$$\Rightarrow \quad n = 6$$

and $\quad {}^6C_3\, m^3 = \frac{5}{2}$

$$\Rightarrow \quad \frac{6.5.4}{3.2}m^3 = \frac{5}{2}$$

$$\Rightarrow \quad m^3 = \frac{1}{8}$$

$$\Rightarrow \quad m = \frac{1}{2}$$

$$\therefore \quad mn = 6\times\frac{1}{2} = 3$$

32. Option (d) is correct.

Given that a, b, c are in GP.

$$\therefore \quad b^2 = ac$$

Now, for $ax^2+bx+c = 0$

$$D = b^2-4ac$$

$$= ac-4ac = -3ac < 0$$

$\therefore$ Roots are imaginary

Let $\quad a = 2, b = 4, c = 8$

$$\therefore \quad x^2+2x+4 = 0$$

$$\Rightarrow \quad x = \frac{-2\pm\sqrt{4-16}}{2}$$

$$= -1\pm\sqrt{3}$$

$$\text{Ratio of roots} = \frac{-1+\sqrt{3}}{-1-\sqrt{3}} = \frac{\omega}{\omega^2} = \frac{1}{\omega}$$

$$\text{Product of roots} = \frac{(-1+\sqrt{3})}{2}\frac{(-1-\sqrt{3})}{2}\times 4$$

$$= \omega.\omega^2.4$$

$$= 4 = \frac{b^2}{a^2}$$

33. Option (c) is correct.

Let n be an integer and $n = 0$

$\therefore \quad x^2 + mn = 0$

$\Rightarrow \quad x(x + m) = 0$

$\Rightarrow \quad x = 0, x = -m$

Since n is integer therefore m is also integer.

34. Option (a) is correct.

$(x + y)^{2n+1}.(x - y)^{2n+1} = (x^2 - y^2)^{2n+1}$

Middle term

$$\frac{2n+2}{2}, \frac{2n+4}{2} = (n+1)^{\text{th term}}, (n+2)^{\text{th term}}$$

$$T_{n+1} = {}^{2n+1}C_n(x^2)^{n+1}(y^2)^n$$

$$T_{n+2} = {}^{2n+1}C_{n+1}(x^2)^n(y^2)^{n+1}$$

$${}^{2n+1}C_n(x^2)^{n+1}(y^2)^n = {}^{2n+1}C_{n+1}(x^2)^n(y^2)^{n+1}$$

$$\Rightarrow \quad \frac{{}^{2n+1}C_n}{{}^{2n+1}C_{n+1}} = \frac{x^{2n}y^{2n+2}}{x^{2n+2}y^{2n}}$$

$$\Rightarrow \quad \frac{(2n+1)!}{(n+1)!n!}\times\frac{n!(n+1)!}{(2n+1)!} = \frac{y^2}{x^2}$$

$$\Rightarrow \quad \frac{y^2}{x^2} = 1$$

35. Option (c) is correct.

$$n(A) = 5 \text{ and } n(B) = 2$$

$$\text{Number of onto functions} = 2^5 - {}^2C_1(2-1)^5$$

$$= 32 - 2 = 30$$

36. Option (d) is correct.

$$\frac{\sqrt{3}\cos 10^\circ - \sin 10^\circ}{\sin 25^\circ.\cos 25^\circ}$$

$$= \frac{2\left(\frac{\sqrt{3}}{2}.\cos 10^\circ - \frac{1}{2}\sin 10^\circ\right)}{\frac{1}{2}(2\sin 25^\circ.\cos 25^\circ)}$$

$$= \frac{4.\sin(60^\circ - 10^\circ)}{\sin 50^\circ}$$

$$= \frac{4\sin 50^\circ}{\sin 50^\circ}$$

$$= 4$$

37. Option (c) is correct.

$$\sin 9^\circ - \cos 9^\circ = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin 9^\circ - \frac{1}{\sqrt{2}}\cos 9^\circ\right)$$

$$= \sqrt{2}\sin(45^\circ - 9^\circ)$$

$$= \sqrt{2}\sin 36^\circ$$

$$= \sqrt{2}\sqrt{1-\cos^2 36^\circ}$$

$$= \sqrt{2}\sqrt{1-\left(\frac{\sqrt{5}+1}{4}\right)^2}$$

$$\left[\because \cos 36^\circ = \frac{\sqrt{5}+1}{4}\right]$$

$$= \sqrt{2}\,\frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$= \frac{\sqrt{5-\sqrt{5}}}{2}$$

38. Option (d) is correct.

Given that,

$\sin^3 A + \sin^3 B + \sin^3 C = 3\sin A \,.\, \sin B \,.\, \sin C$

$\Rightarrow \sin A + \sin B + \sin C = 0$

$\Rightarrow \quad ak + kb + kc = 0$

$\Rightarrow \quad a + b + c = 0$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & b & c \\ b+c+a & c & a \\ c+a+b & a & b \end{vmatrix}$$

(Applying $C_1 \to C_1 + C_2 + C_3$)

$$= \begin{vmatrix} 0 & b & c \\ 0 & c & a \\ 0 & a & b \end{vmatrix}$$

$$= 0$$

39. Option (c) is correct.

$$\cos^{-1}x = \sin^{-1}x$$

$$\Rightarrow \quad \frac{\pi}{2} - \sin^{-1}x = \sin^{-1}x$$

$$\Rightarrow \quad 2\sin^{-1}x = \frac{\pi}{2}$$

$$\Rightarrow \quad \sin^{-1}x = \frac{\pi}{4}$$

$$\Rightarrow \quad x = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

40. Option (b) is correct.

$$(\sin\theta - \cos\theta)^2 = 2$$

$$\Rightarrow \sin^2\theta + \cos^2\theta - 2\sin\theta.\cos\theta = 2$$

$$\Rightarrow \quad \sin 2\theta = -1$$

$\therefore$ Two solutions.

41. Option (c) is correct.

$$\frac{\cos A+\cos B}{\cos\left(\frac{A-B}{2}\right)} = \frac{2\cos\frac{A+B}{2}.\cos\frac{A-B}{2}}{\cos\left(\frac{A-B}{2}\right)}$$

$$= 2\cos\left(\frac{\pi-C}{2}\right)$$

$$= 2\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)$$

$$= 2\sin\frac{C}{2}$$

$= 2 \sin 30°$

$$= 2\times\frac{1}{2}$$

$= 1$

42. Option (c) is correct.

$$2\cot^{-1}3 = 2\tan^{-1}\frac{1}{3} = \tan\frac{\frac{2}{3}}{1-\frac{1}{9}}$$

$$= \tan^{-1}\left(\frac{2}{3}\times\frac{9}{8}\right) = \tan^{-1}\frac{3}{4}$$

So, $\sqrt{15+\cot^2\left(\frac{\pi}{4}-2\cot^{-1}3\right)}$

$$= \sqrt{15+\cot^2\left(\tan^{-1}1-\tan^{-1}\frac{3}{4}\right)}$$

$$= \sqrt{15+\cot^2\left[\tan^{-1}\left(\frac{1-\frac{3}{4}}{1+\frac{3}{4}}\right)\right]}$$

$$= \sqrt{15+\cot^2\left(\tan^{-1}\frac{1}{7}\right)}$$

$$= \sqrt{15+\cot^2(\cot^{-1}7)}$$

$$= \sqrt{15+49}$$

$= 8$

43. Option (b) is correct.

sin 10° . sin 50° + sin 50° . sin 250° + sin 250° . sin 10°

$= \frac{1}{2}$(cos 40° – cos 60° + cos 200° – cos 300° + cos 240° – cos 260°)

$$= \frac{1}{2}\begin{bmatrix}\cos 40°-\frac{1}{2}+\cos(180+20)°-\cos(360-60)° \\ +\cos(180+60)°-\cos(180+80)°\end{bmatrix}$$

$$= \frac{1}{2}\left[\cos 40°-\frac{1}{2}-\cos 20°-\cos 60°-\cos 60°+\cos 80°\right]$$

$$= \frac{1}{2}\left[\cos 40°-\cos 20°+\cos 80°-\frac{1}{2}-\frac{1}{2}-\frac{1}{2}\right]$$

$$= \frac{1}{2}\left[-2\sin 30°.\sin 10°+\cos(90°-10°)-\frac{3}{2}\right]$$

$$= \frac{1}{2}\left[-\sin 10°+\sin 10°-\frac{3}{2}\right]$$

$$= -\frac{3}{4}$$

44. Option (b) is correct.

$$\tan^{-1}\left(\frac{a}{b}\right)-\tan^{-1}\left(\frac{a-b}{a+b}\right)$$

$$= \tan^{-1}\left(\frac{a}{b}\right)-\tan^{-1}\left(\frac{\frac{a}{b}-1}{1+\frac{a}{b}.1}\right)$$

$$= \tan^{-1}\frac{a}{b}-\tan^{-1}\frac{a}{b}+\tan^{-1}1$$

$$= \frac{\pi}{4}$$

45. Option (b) is correct.

For real roots

$(\cos\beta)^2 - 4\sin\beta(\cos\beta - 1) \geq 0$

Since $-1 \leq \cos\beta \leq 1$ and $\sin\beta \geq 0$ for $\beta[\in \{0, \pi]$

So, it is only possible when, $\cos\beta - 1 \geq 0$

46. Option (c) is correct.

$\cos 2A + \cos 2B + \cos 2C$

$= 1 - 2\sin^2 A + 1 - 2\sin^2 B + 1 - 2\sin^2 C$

$= 3 - 2(\sin^2 A + \sin^2 B + \sin^2 C)$

$$= 3-2\left[\left(\frac{16}{65}\right)^2+\left(\frac{63}{65}\right)^2+0\right] = 1$$

[Since, $16^2 + 63^2 = 65^2$]

47. Option (b) is correct.

$$\because \qquad \alpha+\beta = \frac{5\pi}{4}$$

$$\tan(\alpha+\beta) = \tan\frac{5\pi}{4}$$

$$\frac{\tan\alpha+\tan\beta}{1-\tan\alpha.\tan\beta} = \tan\left(\pi+\frac{\pi}{4}\right) = \tan\frac{\pi}{4}$$

$$\frac{\tan\alpha+\tan\beta}{1-\tan\alpha.\tan\beta} = 1$$

$\tan\alpha + \tan\beta + \tan\alpha.\tan\beta = 1$

Since, $f(\theta) = \dfrac{1}{1+\tan\theta}$

$\therefore$ $f(\alpha).f(\beta) = \dfrac{1}{1+\tan\alpha}\times\dfrac{1}{1+\tan\beta}$

$$= \frac{1}{1+\tan\alpha+\tan\beta+\tan\alpha.\tan\beta}$$

$$= \frac{1}{1+1}$$

$$= \frac{1}{2}$$

48. Option (b) is correct.

$\tan\alpha + \tan\beta = 6$

$\tan\alpha . \tan\beta = 8$

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha.\tan\beta}$$

$$= \frac{6}{1-8} = \frac{6}{-7}$$

$$\cos(2\alpha+2\beta) = \cos 2(\alpha+\beta)$$

$$= \frac{1-\tan^2(\alpha+\beta)}{1+\tan^2(\alpha+\beta)}$$

$$= \frac{1-\frac{36}{49}}{1+\frac{36}{49}}$$

$$= \frac{13}{85}$$

49. Option (c) is correct.

$\tan(90-25)^\circ + 2 - 2\tan 40^\circ - \tan 25^\circ$

$= \cot 25^\circ - \tan 25^\circ - 2\tan 40^\circ + 2$

$= \dfrac{\cos^2 25 - \sin^2 25}{\sin 25.\cos 25} - 2\tan 40^\circ + 2$

$= \dfrac{2\cos 50^\circ}{\sin 50^\circ} - 2\tan 40^\circ + 2$

$= 2\cot 50^\circ - 2\tan 40^\circ + 2$

$= 2\tan 40^\circ - 2\tan 40^\circ + 2 = 2$

50. Option (a) is correct.

If triangle is acute angled triangle then

$\cot A. \cot B. \cot C > 0$

So, statement 1 is correct.

If triangle is obtuse angled triangle, then one of A, B, C is obtuse i.e., one of the $\tan A$, $\tan B$ and $\tan C$ is –ve

$\therefore \tan A . \tan B . \tan C < 0$

So, statement 2 is wrong.

51. Option (a) is correct.

Given the circle of the equation is

$x^2 + y^2 + 2x + 6y + 1 = 0$

To find the centre (a, b) and the radius c of a circle equation, we need to rewrite the equation in the form $(x-h)^2 + (y-k)^2 = r^2$, where (h, k) is the centre of the circle and r is its radius.

Let $x^2 + y^2 + 2x + 6y + 1 = 0$

First complete the square to get the equation in the desired form:

$(x^2 + 2x + 1) - 1 + (y^2 + 6y + 9) - 9 + 1 = 0$

$\Rightarrow (x+1)^2 - 1 + (y+3)^2 - 9 + 1 = 0$

$\Rightarrow (x+1)^2 + (y+3)^2 - 9 = 0$

$\Rightarrow (x+1)^2 + (y+3)^2 = 9$

$\Rightarrow [x-(-1)]^2 + [y-(-3)]^2 = (3)^2$

Now we can see that the centre of the circle is at (–1, –3) and the radius is 3.

$\therefore$ $a = -1, b = -3$ and $c = 3$

$\therefore$ $a^2 + b^2 + c^2 = (-1)^2 + (-3)^2 + (3)^2$

$= 1 + 9 + 9$

$= 19$

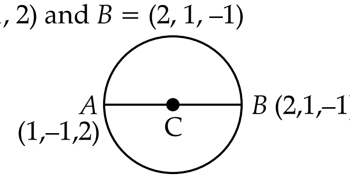
52. Option (a) is correct.

The general equation $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ represents a sphere with centre $(-u, -v, -w)$ and radius $r = \sqrt{u^2+v^2+w^2-d}$

If B is the mid-point of the line segment AC where $A = (x_1, y_1, z_1)$ and $C = (x_2, y_2, z_2)$ then

$$B = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

Given: Equation of sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz - 1 = 0$ with AB as its diameter and $A = (1, -1, 2)$ and $B = (2, 1, -1)$



$$C = \left(\frac{1+2}{2}, \frac{1-1}{2}, \frac{-1+2}{2}\right)$$

$$= \left(\frac{3}{2}, 0, \frac{1}{2}\right)$$

As we know that centre of the sphere is given by: $(-u, -v, -w)$

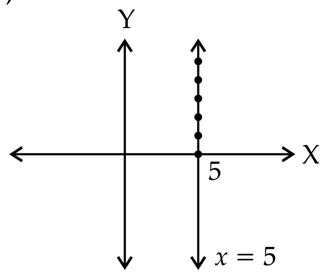
So compare that we get

$$\left(\frac{3}{2}, 0, \frac{1}{2}\right) = (-u, -v, -w)$$

$$u = -\frac{3}{2}, v = 0, w = -\frac{1}{2}$$

$$\therefore \quad u + v + w = -\frac{3}{2} + 0 - \frac{1}{2}$$
$$= -\frac{4}{2}$$
$$= -2$$

53. Option (d) is correct.



$x = 5$ represents a line that is parallel to y-axis and cuts the x-axis at 5 units from the origin.

It is given that $x = 5$, this implies that the y-coordinate is zero.

Here the infinite number of points i.e., $(5, y)$ represent the equation $x = 5$ on the xy-plane.

54. Option (b) is correct.

We know the plane equation is $ax + by + cz = d$.

Given the plane equation is

$$2x - 3y + 6z + 4 = 0$$
$$\Rightarrow \quad 2x - 3y + 6z = -4$$

Comparing with plane equation we get

$$a = 2, b = -3, c = 6, d = -4$$
$$\therefore \quad \sqrt{a^2 + b^2 + c^2} = \sqrt{(2)^2 + (-3)^2 + (6)^2}$$
$$= \sqrt{4 + 9 + 36} = \sqrt{49}$$
$$= 7$$

Here we need to find the direction cosine of a normal to the plane i.e., $< l, m, n >$

Direction cosines of normal to plane

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$
$$= \frac{2}{\sqrt{49}} = \frac{2}{7}$$
$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$
$$= \frac{-3}{\sqrt{49}} = \frac{-3}{7}$$
$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$
$$= \frac{6}{\sqrt{49}} = \frac{6}{7}$$

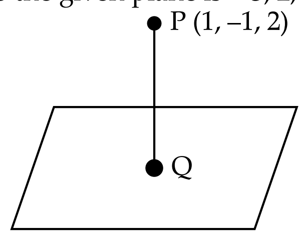
$$\therefore \quad <l, m, n> = <\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}>$$

$$\therefore 49(7l^2 + m^2 - n^2) =$$
$$49\left[7\left(\frac{2}{7}\right)^2 + \left(-\frac{3}{7}\right)^2 - \left(\frac{6}{7}\right)^2\right]$$
$$= 49\left(\frac{28}{49} + \frac{9}{49} - \frac{36}{49}\right)$$
$$= 49\left(\frac{37 - 36}{49}\right)$$
$$= 1$$

55. Option (c) is correct.

Let the line from point be P(1, –1, 2) and meet the plane at point Q.

Direction ratios of the line from the point (1, –1, 2) to the given plane is <3, 2, 2>



So the equation of the line passing through P and with direction ratios will be:

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{2} = \lambda$$

Coordinates of any points on the line PQ are

$x = 3\lambda + 1, y = 2\lambda - 1, z = 2\lambda + 2$

Now, since Q lies on the plane so it must satisfy the equation of the plane.

i.e., $\quad x + 2y + 3z = 18$

$\therefore 3\lambda + 1 + 4\lambda - 2 + 6\lambda + 6 = 18$

$\Rightarrow \quad 13\lambda + 5 = 18$

$\Rightarrow \quad \lambda = 1$

So, the coordinates of Q are:

$(3 + 1, 2 - 1, 2 + 2) = (4, 1, 4)$

56. Option (d) is correct.

Given that plane passing through (1, 0, 0), (0, 1, 0) and (0, 0, 1).

First we will find the equation of plane

$$\vec{a} = \hat{i}, \vec{b} = \hat{j}, \vec{c} = \hat{k}$$

Let $\quad \hat{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$(\vec{r} - \vec{a}).[(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$$

$$\therefore \quad \vec{b} - \vec{a} = \hat{j} - \hat{i} \text{ and } \vec{c} - \vec{a} = \hat{k} - \hat{i}$$

$$\therefore \quad (\vec{b}-\vec{a})\times(\vec{c}-\vec{a}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$$= \hat{i}+\hat{j}+\hat{k}$$

$$\therefore \quad (\vec{r}-\vec{a}) = (x\hat{i}+y\hat{j}+z\hat{k})-(\hat{i})$$

$$= (x-1)\hat{i}+y\hat{j}+z\hat{k}$$

We have,

$$[(x-1)\hat{i}+y\hat{j}+z\hat{k}].(\hat{i}+\hat{j}+\hat{k}) = 0$$

$$= (x-1)+y+z = 0$$

$\therefore$ Equation of plane $x + y + z - 1 = 0$

Perpendicular distance from origin to the plane

$$= \frac{|-1|}{\sqrt{(1)^2+(1)^2+(1)^2}}$$

$$= \frac{1}{\sqrt{3}} \text{ unit}$$

Here, $\quad p = \dfrac{1}{\sqrt{3}}$

$$\therefore \quad 3p^2 = 3\times\frac{1}{3} = 1$$

57. Option (b) is correct.

We have, $l + 2m + n = 0$ ...(i)

and $2l - 2m + 3n = 0$ ...(ii)

From (i) we have,

$$l = -2m - n \quad \text{...(iii)}$$

Put (iii) in (ii)

$$2(-2m-n) - 2m + 3n = 0$$
$$-4m - 2n - 2m + 3n = 0$$
$$-6m + n = 0$$
$$n = 6m$$

Now we have $\quad l = -2m - n$

$$= -2m - 6m = -8m$$

$$\frac{l}{-8m} = \frac{m}{m} = \frac{n}{6m}$$

$$\therefore \quad \frac{l}{-8} = \frac{m}{1} = \frac{n}{6}$$

$$= \sqrt{\frac{l^2+m^2+n^2}{(-8)^2+(1)^2+(6)^2}}$$

$$= \frac{1}{\sqrt{101}}$$

$$\therefore \quad l = \frac{-8}{\sqrt{101}}, m = \frac{1}{\sqrt{101}},$$

$$n = \frac{6}{\sqrt{101}}$$

$\therefore$ Direction cosines of a line

$$<\frac{-8}{\sqrt{101}}, \frac{1}{\sqrt{101}}, \frac{6}{\sqrt{101}}> = <l, m, n>$$

$$\therefore \quad l^2 + m^2 - n^2 = \frac{64}{101}+\frac{1}{101}-\frac{36}{101}$$

$$= \frac{29}{101}$$

58. Option (b) is correct.

Let the equation of any line passing through the point of intersection of the given line be

$$(x + 2y - 1) + a(2x - y - 1) = 0$$

Reducing the equation to its intercept form

$$x + 2y - 1 + 2ax - ay - a = 0$$

$$\Rightarrow \quad (1 + 2a)x + (2 - a)y = 1 + a$$

$$\Rightarrow \quad \frac{(1+2a)}{(1+a)}x + \frac{(2-a)}{(1+a)}y = 1$$

Therefore coordinates of A and B, where this line meets the coordinate axis respectively.

$A = \left(\dfrac{1+a}{1+2a}, 0\right)$ on x-axis

$B = \left(0, \dfrac{1+a}{2-a}\right)$ on y-axis

Now we need to find the mid-point of AB by using coordinates of A and B

$$\text{Mid-point of } AB = \left(\frac{1+a}{2+4a}, \frac{1+a}{4-2a}\right)$$

Now, we find the locus of this point by eliminating 'a' between the two expressions.

Where $x = \dfrac{1+a}{2+4a}$ and $y = \dfrac{1+a}{4-2a}$

$$\Rightarrow \quad 2x + 4ax = 1 + a$$

$$\Rightarrow \quad 4ax - a = 1 - 2x$$

$$\Rightarrow \quad a = \frac{1-2x}{4x-1}$$

We get

$$y = \frac{1+\frac{1-2x}{4x-1}}{4-2\left(\frac{1-2x}{4x-1}\right)}$$

$$y = \frac{4x-1+1-2x}{16x-4-2+4x}$$

$$y = \frac{2x}{20x-6}$$

$$\Rightarrow \quad y = \frac{x}{10x-3}$$

$$\therefore \quad 10xy - 3y = x$$

$$\Rightarrow \quad x + 3y = 10xy$$

59. Option (b) is correct.

First we need to find slope of the given line:

$x \cos\theta + y \sin\theta = 9$

Rewrite the equation as follows:

$$y \sin\theta = 9 - x\cos\theta$$

$$\Rightarrow \quad y = \frac{9}{\sin\theta} - \frac{x\cos\theta}{\sin\theta}$$

$$\Rightarrow \quad y = -\frac{x\cos\theta}{\sin\theta} + \frac{9}{\sin\theta} \quad \text{...(i)}$$

$\because$ The general equation of line is:

$$y = mx + c \quad \text{...(ii)}$$

Where (x, y) is the general point on the line. m is the slope of the line, c is the y-intercept.

On comparing equation (i) and (ii), we get

$$m = -\frac{\cos\theta}{\sin\theta}$$

We know that, the slope of perpendicular line are negative inverse of each other.

$\therefore$ The slope m_1 of the required line can be

$$m_1 = -\left(\frac{1}{-\frac{\cos\theta}{\sin\theta}}\right)$$

$$\Rightarrow \quad m_1 = \frac{\sin\theta}{\cos\theta}$$

Also, the line passes through the point $(-\sin\theta, \cos\theta)$. We know that, one point slope from the line is:

$$y - y_1 = m(x - x_1)$$

Where m is the slope and (x_1, y_1) is the given point on the line.

$\therefore$ The equation of the required line is

$$y - \cos\theta = \frac{\sin\theta}{\cos\theta}(x + \sin\theta)$$

$$\Rightarrow \quad \cos\theta\, y - \cos^2\theta = x\sin\theta + \sin^2\theta$$

$$\Rightarrow x\sin\theta - y\cos\theta + \sin^2\theta + \cos^2\theta = 0$$

$$\Rightarrow x\sin\theta - y\cos\theta + 1 = 0$$

60. Option (a) is correct.

We have given P and Q lie on line $y = 2x + 3$

For P, Put $x = a$ then $y = 2a + 3$

For Q, Put $x = b$ then $y = 2b + 3$

Coordinates $P = (a, 2a + 3)$ and $Q = (b, 2b + 3)$

$R = (1, 5)$

From using distance formula, we have

$$PR = \sqrt{(a-1)^2 + (2a+3-5)^2}$$

$$= 2 \quad \text{...(i)}$$

$$QR = \sqrt{(b-1)^2 + (2b+3-5)^2}$$

$$= 2 \quad \text{...(ii)}$$

From (i), we get

$(a-1)^2 + (2a-2)^2 = 4$

$a^2 - 2a + 1 + 4a^2 + 4 - 8a = 4$

$5a^2 - 10a + 1 = 0$

From (ii), we get

$5b^2 - 10b + 1 = 0$

By using quadratic formula

$$a = \frac{10 \pm \sqrt{100-20}}{10}$$

$$\Rightarrow \quad a = 1 \pm \frac{2\sqrt{5}}{5}$$

$$\Rightarrow \quad a = 1 \pm \frac{2}{\sqrt{5}}$$

$$\therefore \quad b = 1 \pm \frac{2}{\sqrt{5}}$$

When $a = 1 + \frac{2}{\sqrt{5}}$ then

Coordinate of $P = (a, 2a + 3)$

$$= \left(1 + \frac{2}{\sqrt{5}}, 2 + \frac{4}{\sqrt{5}} + 3\right)$$

$$= \left(1 + \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right)$$

When $a = 1 - \frac{2}{\sqrt{5}}$ then

Coordinate of $P = (a, 2a + 3)$

$$= \left(1 - \frac{2}{\sqrt{5}}, 2 - \frac{4}{\sqrt{5}} + 3\right)$$

$$= \left(1 - \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$$

Similarly coordinate of $Q = \left(1 - \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$ or

$$= \left(1 + \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right)$$

So we get coordinate of the point P and Q is

$$\left(1 + \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right), \left(1 - \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$$

61. Option (a) is correct.

We have two sides of square lie on the lines

$$2x + y - 3 = 0 \quad ...(i)$$

$$4x + 2y + 5 = 0 \quad ...(ii)$$

Divide (ii) by 2 we get

$$2x+y+\frac{5}{2} = 0 \quad ...(iii)$$

Here $2x + y - 3 = 0$ and $2x+y+\frac{5}{2} = 0$ are parallel lines.

Therefore, length of the side of the square = Distance between parallel side

$$= \frac{|c_1 - c_2|}{\sqrt{a_1^2+b_1^2}}$$

Where $c_1 = -3$ and $c_2 = \frac{5}{2}$

$$(a_1, b_1) = (2, 1)$$

$$\Rightarrow \frac{\left|-3-\frac{5}{2}\right|}{\sqrt{(2)^2+(1)^2}} = \frac{\left|-\frac{11}{2}\right|}{\sqrt{5}}$$

$$= \frac{11}{2\sqrt{5}}$$

Length of the side of square = $\frac{11}{2\sqrt{5}}$ units

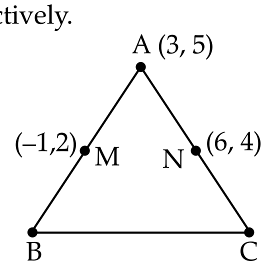
Area of the square = $(\text{side})^2 = \left(\frac{11}{2\sqrt{5}}\right)^2$

$= 6.05$ square units.

62. Option (b) is correct.

Given $A = (3, 5)$ and

Mid-point of sides AB and AC is (–1, 2) and (6, 4) respectively.



Let $B = (x_1, y_1)$ and $C = (x_2, y_2)$

By using mid-point formula we will find the point B and C

$$M = (-1, 2) = \left(\frac{x_1+3}{2}, \frac{y_1+5}{2}\right)$$

$$N = (6, 4) = \left(\frac{x_2+3}{2}, \frac{y_2+5}{2}\right)$$

By comparing we get

$$\frac{x_1+3}{2} = -1, \frac{y_1+5}{2} = 2$$

$$\Rightarrow x_1 = -5, y_1 = -1$$

and $$\frac{x_2+3}{2} = 6, \frac{y_2+5}{2} = 4$$

$$\Rightarrow x_2 = 9, y_2 = 3$$

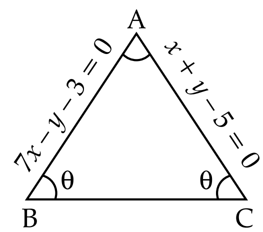
$\therefore$ $B = (-5, -1)$ and $C(9, 3)$

Centroid of $\Delta ABC = \left(\frac{3-5+9}{3}, \frac{5-1+3}{3}\right)$

$$= \left(\frac{7}{3}, \frac{7}{3}\right)$$

63. Option (b) is correct.

Given that ABC is an acute angled isosceles triangle



First we find slope of $AB(m_1) = 7$

and slope of $AC(m_2) = -1$

We know that $\tan A = \left|\frac{m_1 - m_2}{1+m_1m_2}\right|$

$$= \left|\frac{7-(-1)}{1+(-7)}\right|$$

$$= \frac{8}{6} = \frac{4}{3}$$

$$\Rightarrow \tan A = \frac{4}{3}$$

$$A = \tan^{-1}\frac{4}{3}$$

We know that sum of all the angle of triangle $= \pi$

i.e., $$\theta+\theta+\tan^{-1}\frac{4}{3} = \pi$$

$$\Rightarrow 2\theta = \pi - \tan^{-1}\frac{4}{3}$$

$$\Rightarrow \theta = \frac{\pi}{2}-\frac{1}{2}\tan^{-1}\frac{4}{3}$$

Apply cot both side, we get

$$\cot\theta = \cot\left(\frac{\pi}{2}-\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$$

$$\cot\theta = \tan\left(\frac{\tan^{-1}\frac{4}{3}}{2}\right) \quad ...(i)$$

Let $\tan^{-1}\frac{4}{3} = \alpha$

We know that $\tan\frac{\alpha}{2} = \text{cosec}\,\alpha - \cot\alpha$

From (i) we get

$$\cot\theta = \text{cosec}\left(\tan^{-1}\frac{4}{3}\right)-\cot\left(\tan^{-1}\frac{4}{3}\right) \quad ...(ii)$$

We have $\tan^{-1}\frac{4}{3} = \alpha$

$$\tan\alpha = \frac{4}{3}$$

$$\Rightarrow \quad \cot\alpha = \frac{3}{4} \text{ and } \text{cosec}\,\alpha = \frac{5}{4}$$

Put in equation (ii) we get

$$\cot\theta = \text{cosec}\,\alpha - \cot\alpha = \frac{5}{4}-\frac{3}{4} = \frac{2}{4} = \frac{1}{2}$$

64. Option (c) is correct.

Equation of parabola is $y^2 = 8x$

$\therefore \quad a = 2$

We know that focal distance of point P(x_1, y_1) is $x_1 + a$

$\therefore$ Focal distance from point $(6, 4\sqrt{3})$ is $6 + 2$

$= 8$

So, statement 1 is correct.

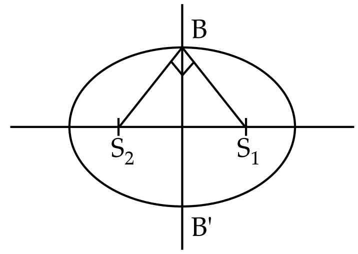
Distance of point $P(6, 4\sqrt{3})$ from directrix

$$PF = \sqrt{(6-2)^2+(4\sqrt{3}-0)^2} = \sqrt{16+48} = 8$$

So, statement 2 is also correct.

65. Option (d) is correct.

Let ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$



It's foci are $S_1(ae, 0)$ and $S_2(-ae, 0)$ and the extremity of the minor axis is $B(0, b)$

Then slope of $BS_2 = \frac{b-0}{0-(-ae)} = \frac{b}{ae}$

and slope of $BS_1 = \frac{b-0}{0-ae} = \frac{-b}{ae}$

$\therefore$ The product of slopes, $m_{BS_2} \times m_{BS_1} = -1$

[$\because$ S_1B and S_2B are perpendicular]

$$\Rightarrow \quad \frac{-b}{ae}\times\frac{b}{ae} = -1$$

$$b^2 = a^2e^2$$

$$\Rightarrow \quad e^2 = \frac{b^2}{a^2} \quad ...(i)$$

As we know that $e = \frac{c}{a}$, where

$$c = \sqrt{a^2-b^2}$$

$$\Rightarrow \quad e^2 = 1-\frac{b^2}{a^2}$$

$$= 1-e^2 \quad \text{[from (i)]}$$

$$\Rightarrow \quad 2e^2 = 1$$

$$\Rightarrow \quad e^2 = \frac{1}{2}$$

$$\Rightarrow \quad e = \frac{1}{\sqrt{2}}$$

66. Option (d) is correct.

We know that $\vec{a} = \hat{i}-\hat{j}+\hat{k}$ and

$\vec{b} = \hat{i}+2\hat{j}-\hat{k}$

We need to find $\vec{a}\times(\vec{b}\times\vec{a})$

We know that

$$\vec{a}\times(\vec{b}\times\vec{a}) = (\vec{a}.\vec{a})\vec{b}-(\vec{a}.\vec{b})\vec{a}$$

$$= [(\hat{i}-\hat{j}+\hat{k}).(\hat{i}-\hat{j}+\hat{k})]\vec{b}-[(\hat{i}-\hat{j}+\hat{k}).(\hat{i}+2\hat{j}-\hat{k})]\vec{a}$$

$$= (1+1+1)\vec{b}-(1-2-1)\vec{a}$$

$$= 3(\hat{i}+2\hat{j}-\hat{k})+2(\hat{i}-\hat{j}+\hat{k})$$

$$= 5\hat{i}+4\hat{j}-\hat{k} \quad ...(i)$$

Given, $\vec{a}\times(\vec{b}\times\vec{a}) = \alpha\hat{i}-\beta\hat{j}+\gamma\hat{k}$

Compare that with (i), we get

$$\alpha = 5, \beta = -4, \gamma = -1$$

$\therefore \quad \alpha + \beta + \gamma = 5 - 4 - 1 = 0$

67. Option (a) is correct.

Let a vector $\vec{a}$ have (a_1, a_2, a_3) components

$\Rightarrow a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$\because \quad |\vec{a}| = 2$

Also, it is given that $\vec{a}$ make angle with $\frac{\pi}{3}$ with $2\hat{i}$, $\frac{\pi}{4}$ with $3\hat{j}$ and an acute angle θ with $4\hat{k}$

Then, we have

$$\vec{a}.2\hat{i} = |\vec{a}||2\hat{i}|\cos\frac{\pi}{3}$$

$$(a_1\hat{i}+a_2\hat{j}+a_3\hat{k}).2\hat{i} = 2.2\times\frac{1}{2}$$

$$2a_1 = 2$$

$$a_1 = 1$$

$$\therefore \quad \vec{a}.3\hat{j} = |\vec{a}||3\hat{j}|\cos\frac{\pi}{4}$$

$$3a_2 = 2\times3\times\frac{1}{\sqrt{2}}$$

$$a_2 = \sqrt{2}$$

and $\quad \vec{a}.4\hat{k} = |\vec{a}||4\hat{k}|\cos\theta$

$$4a_3 = 2\times4\cos\theta$$

$$\cos\theta = \frac{a_3}{2}$$

Now, $\quad |\vec{a}| = 2$

$$\sqrt{a_1^2+a_2^2+a_3^2} = 2$$

Square both side, we get

$$a_1^2 + a_2^2 + a_3^2 = 4$$

Put the value of $a_1 = 1$ and $a_2 = \sqrt{2}$

$$(1)^2 + (\sqrt{2})^2 + a_3^2 = 4$$

$\Rightarrow \quad a_3^2 = 4 - 1 - 2$

$\Rightarrow \quad a_3^2 = 1$

$\Rightarrow \quad a_3^2 = \pm1$

When $\quad a_3 = -1$ then $\cos\theta = \frac{-1}{2}$

$\Rightarrow \quad \theta = \frac{2\pi}{3} > \frac{\pi}{2} \quad$ (contradict)

$\because$ θ is acute angle with $4\hat{k}$

So $\quad a_3 = 1$

Hence, the components of $\vec{a} = (1, \sqrt{2}, 1)$

68. Option (b) is correct.

Statement 1 is wrong.

We know that $\vec{\tau} = \vec{r}\times\vec{F}$

The moment of force about a point is dependent of application of force

Statement 2 is correct.

The moment of force about a line is a vector quantity.

Cross product of two vector is again a vector

$$\underset{\text{vec}}{\vec{\tau}} = \underset{\text{vec}}{\vec{r}}\times\underset{\text{vec}}{\vec{F}}$$

69. Option (a) is correct.

Given

$$(\vec{r}.\hat{i})(\vec{r}\times\hat{i})+(\vec{r}.\hat{j})(\vec{r}\times\hat{j})+(\vec{r}.\hat{k})(\vec{r}\times\hat{k}) \quad \text{...(i)}$$

We know that

$$\vec{r} = a\hat{i}+b\hat{j}+c\hat{k}$$

$$(\vec{r}.\hat{i}) = (a\hat{i}+b\hat{j}+c\hat{k}).\hat{i} = a$$

$$(\vec{r}.\hat{j}) = b$$

$$(\vec{r}.\hat{k}) = c$$

and $\quad \vec{r}\times\hat{i} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a & b & c\\ 1 & 0 & 0\end{vmatrix}$

$$= -\hat{j}(-c)-b\hat{k}$$

$$= c\hat{j}-b\hat{k}$$

Similarly $\quad \vec{r}\times\hat{j} = -c\hat{i}+a\hat{k}$ and

$$\vec{r}\times\hat{k} = b\hat{i}-a\hat{j}$$

Now put all these values in equation (i), we get

$$a(c\hat{j}-b\hat{k})+b(-c\hat{i}+a\hat{k})+c(b\hat{i}-a\hat{j})$$

$$= ac\hat{j}-ab\hat{k}-cb\hat{i}+ab\hat{k}+bc\hat{i}-ac\hat{j}$$

$$= \vec{0}$$

70. Option (b) is correct.

The angle between $\vec{a}$ and $\vec{a}-\vec{b}$ is given by

$$\cos\theta = \frac{\vec{a}.(\vec{a}-\vec{b})}{|\vec{a}||\vec{a}-\vec{b}|}$$

Now, $\quad \vec{a}.(\vec{a}-\vec{b}) = \vec{a}.\vec{a}-\vec{a}.\vec{b}$

$\therefore \quad \vec{a}.\vec{a} = |\vec{a}|^2$

Given that magnitude of $\vec{a}$ and $\vec{b}$ is 4

i.e., $\quad |\vec{a}| = |\vec{b}| = 4$

$\therefore \quad |\vec{a}|^2 = 4^2 = 16$

and $\quad \vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\frac{\pi}{3}$

$$= 4.4.\frac{1}{2}$$

$$= 8$$

$$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 - |\vec{b}|^2 - \vec{a}.\vec{b} - \vec{b}.\vec{a}$$

$$= 16 + 16 - 8 - 8$$

$$= 16$$

$$\cos\theta = \frac{16-8}{4.\sqrt{16}} = \frac{8}{4.4} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

71. Option (a) is correct.

Given, D.E: $\frac{dy}{dx} = x$

$\Rightarrow \quad \int dy = \int x\,dx$

$\Rightarrow \quad y = \frac{x^2}{2} + C$

$\Rightarrow \quad y_1(x) = \frac{x^2}{2} + C_1 \quad$ ...(i)

and $\quad y_2(x) = \frac{x^2}{2} + C_2 \quad$ ...(ii)

For $\quad y_1(0) = 0$ in equation (i)

$\Rightarrow \quad 0 = 0 + C_1$

$\Rightarrow \quad C_1 = 0$

$\Rightarrow \quad y_1(x) = \frac{x^2}{2} \quad$ ...(iii)

For $\quad y_2(0) = 0$

$\Rightarrow \quad 4 = 0 + C_2 \quad$ [from (ii)]

$\Rightarrow \quad C_2 = 4$

$\Rightarrow \quad y_2(x) = \frac{x^2}{2} + 4 \quad$ ...(iv)

So, for number of points of intersection for $y_1(x)$ and $y_2(x)$ solving (iii) and (iv)

$\frac{x^2}{2} = \frac{x^2}{2} + 4 \quad$ (Not possible)

as $\quad 0 \neq 4$

$\therefore$ No point of intersection.

72. Option (c) is correct.

Given, $\quad y = e^x(a\cos x + b\sin x) \quad$ ...(i)

$\frac{dy}{dx} = e^x(-a\sin x + b\cos x) + (a\cos x + b\sin x)e^x$

$\frac{dy}{dx} = e^x(-a\sin x + b\cos x) + y \quad$ [using (i)]...(ii)

Again differentiate w.r.t. to x,

$\frac{d^2y}{dx^2} = -e^x(a\cos x + b\sin x) + (-a\sin x + b\cos x)e^x + \frac{dy}{dx}$

$\frac{d^2y}{dx^2} = -y + \frac{dy}{dx} - y + \frac{dy}{dx} \quad$ [Using (i) and (ii)]

$\frac{d^2y}{dx^2} - \frac{2dy}{dx} + 2y = 0$ is the required differential equation.

73. Option (d) is correct.

$f(x) = ax - b \quad$ ...(i)

and $\quad g(x) = cx + d \quad$ ...(ii)

$f[g(x)] = g[f(x)]$

$\Rightarrow \quad a[g(x)] - b = c[f(x)] + d$

$\Rightarrow \quad a(cx + d) - b = c(ax - b) + d$

[using (i) and (ii)]

$\Rightarrow \quad acx + ad - b = acx - bc + d$

$\Rightarrow \quad ad - b = d - bc$

$\Rightarrow \quad ad - b + bc + d = d + d$

$\therefore \quad f(d) + g(b) = 2d$

[$\because$ $(d) = ad - b$ and $g(b) = bc + d$]

74. Option (b) is correct.

Let $\quad I = \int_{-1}^{1}(3\sin x - \sin 3x)\cos^2 x\,dx$

$I = \int_{-1}^{1}(3\sin x - 3\sin x + 4\sin^3 x)\cos^2 x\,dx$

[$\because$ $\sin 3x = 3\sin x - 4\sin^3 x$]

$I = \int_{-1}^{1} 4\sin^3 x\cos^2 x\,dx$

$= \int_{-1}^{1} f(x)dx$

Here, $f(x) = 4\sin^3 x\cos^2 x$ is an odd function as $f(-x) = -f(x)$

$\Rightarrow \quad I = \int_{-1}^{1} 4\sin^2 x\cos^2 x\,dx = 0$

[$\because$ $\int_{-a}^{a} f(x)dx = 0$, if $f(-x) = -f(x)$]

75. Option (d) is correct.

Given

$$\left\{2 - \left(\frac{dy}{dx}\right)^2\right\}^{0.6} = \frac{d^2y}{dx^2}$$

Removing decimal power/fraction power both sides

$$\left\{2-\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{5}} = \frac{d^2y}{dx^2}$$

Raise power of 5 both sides,

$$\left\{2-\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{5}\times 5} = \left(\frac{d^2y}{dx^2}\right)^5$$

$$\Rightarrow \left\{2-\left(\frac{dy}{dx}\right)^2\right\}^{3} = \left(\frac{d^2y}{dx^2}\right)^5$$

Now differential equation is free from decimal or fraction power.

So, Order $= 2$

Degree $= 5$

76. Option (a) is correct.

$$\frac{dy}{dx} = 2e^x y^3,$$

$$\Rightarrow \frac{dy}{y^3} = 2e^x\,dx$$

Integrating both sides w.r.t. x

$$\frac{-1}{2y^2} = 2e^x + C \qquad \text{...(i)}$$

$$y(0) = \frac{1}{2}, \qquad \text{Substitute in (i)}$$

$$-2 = 2 + C$$

$$\Rightarrow C = -4$$

The solution is:

$$\frac{-1}{2y^2} = 2e^x - 4$$

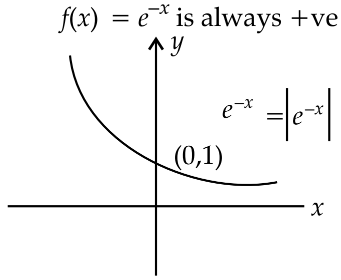
$$\Rightarrow 4y^2(e^x) - 8y^2 = -1$$

$$\Rightarrow 4y^2(2 - e^x) = 1$$

77. Option (d) is correct.

Given, $p = \int_a^b f(x)dx$ and $q = \int_a^b |f(x)|\,dx$

Since $f(x) = e^{-x}$ is always +ve $\forall\, x \in$ R



$$|f(x)| = |e^{-x}| = e^{-x}$$
$$= f(x)$$

$$\therefore \int_a^b f(x)dx = \int_a^b |f(x)|\,dx$$

$$\therefore p = q$$

78. Option (a) is correct.

Let $$I = \int_0^{\frac{\pi}{2}} \frac{a+\sin x}{2a+\sin x+\cos x}dx \qquad \text{...(i)}$$

$$I = \int_0^{\frac{\pi}{2}} \frac{a+\cos x}{2a+\cos x+\sin x}dx \qquad \text{...(ii)}$$

$$\left[\int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

Adding equation (i) and (ii)

$$2I = \int_0^{\frac{\pi}{2}} \frac{2a+\sin x+\cos x}{2a+\sin x+\cos x}dx$$

$$= \int_0^{\frac{\pi}{2}} dx = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

79. Option (d) is correct.

Let $$f(x) = \frac{16x^3}{3} - 4bx^2 + x$$

$$f'(x) = 16x^2 - 8bx + 1$$

Since $f(x)$ his neither maximum nor minimum, then $f'(x) \neq 0$

$f'(x) > 0$ or $f'(x) < 0$

(but here $a > 0$ for $f'(x)$ so not possible)

for $f'(x) > 0$

Discriminant of $f'(x)$ is D < 0 and $a > 0$

$$D = 64b^2 - 64 < 0$$

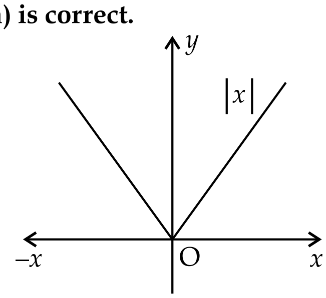
$b^2 - 1 < 0$ and b is non-negative as $b \geq 0$

$b \in (-1, 1)$ and $b \geq 0$

$\therefore b \in [0, 1)$

$\therefore 0 \leq b < 1$

80. Option (a) is correct.



$$f(x) = \frac{1}{\sqrt{|x|-x}};$$

Domain of $f(x)$, $|x| - x > 0$

i.e., $|x| < x$

$\Rightarrow x \in (-\infty, 0)$

$$g(x) = \frac{1}{\sqrt{x-|x|}}$$

Domain of $g(x) = x - |x| > 0$

$\Rightarrow \qquad |x| > x$

(Not possible as $|x|$ is always greater than or equal to x)

Solution for Q.no. 81 and 82:

Let $\qquad I = \int \frac{3\cos x + 4\sin x}{2\cos x + 5\sin x} dx$

$3 \cos x + 4 \sin x = A(2 \cos x + 5 \sin x) + B\left[\frac{d}{dx}(2\cos x + 5\sin x)\right]$

$3 \cos x + 4 \sin x$

$= A(2 \cos x + 5 \sin x) + B(-2 \sin x + 5 \cos x)$

Comparing coefficient of $\sin x$ and $\cos x$

$$3 = 2A + 5B \qquad \text{...(i)}$$

$$4 = 5A - 2B \qquad \text{...(ii)}$$

Solving (i) and (ii)

$$A = \frac{26}{29}, B = \frac{7}{29}$$

$$I = \int \frac{\frac{26}{29}(2\cos x + 5\sin x) + \frac{7}{29}(-2\sin x + 5\cos x)}{2\cos x + 5\sin x} dx$$

$$I = \int \frac{26}{29} dx + \frac{7}{29}\int \frac{(-2\sin x + 5\cos x)}{2\cos x + 5\sin x} dx$$

$$I = \frac{26}{29}x + \frac{7}{29}\ln(2\cos x + 5\sin x) + c$$

$$\alpha = 26, \beta = 7 \quad \text{(on comparing)}$$

81. Option (d) is correct.

$$\alpha = 26$$

82. Option (a) is correct.

$$\beta = 7$$

Solution for Q.no. 83 and 84:

83. Option (d) is correct.

$$f(x) = \frac{x}{\ln(x)} (x > 1)$$

$$f'(x) = \frac{\ln x - 1}{[\ln(x)]^2}$$

– ve $\qquad$ + ve

1 $\qquad$ e

$f'(x)$ is +ve, $x \in (e, \infty)$ i.e., $f(x)$ is increasing in interval $(e, \infty) \Rightarrow 1$ is correct.

$f'(x)$ is –ve, $\forall x \in (1, e)$ i.e., $f(x)$ is decreasing in the interval $(1, e) \Rightarrow 2$ is correct.

As $\ln x$ is an increasing function

$$7^9 > 9^7$$

$$\ln 7^9 > \ln 9^7$$

$$9 \ln 7 > 7 \ln 9$$

$\Rightarrow$ statement 3 is correct

$\therefore$ Statement 1, 2 and 3 are correct.

84. Option (d) is correct.

$$f''(x) = \frac{(\ln x)^2 \times \frac{1}{x} - (\ln x - 1) \times 2\frac{\ln x}{x}}{(\ln x)^4}$$

$$= \frac{(\ln x)^2 - 2(\ln x)^2 + 2\ln x}{x(\ln x)^4}$$

$$\Rightarrow \qquad f''(e) = \frac{1-2+2}{e \times 1} = \frac{1}{e} \ (> 0)$$

$\Rightarrow$ Statement 1 is correct

Since $\quad f''(e) > 0$ $\qquad$ (true)

$f(x)$ attains local minima at $x = e$.

$\Rightarrow$ Statement 2 is correct

A local minimum value occurs at $x = e$.

$$f(x) \text{ at } x = e, \text{ is } f(e) = \frac{e}{\ln e} = e$$

$\Rightarrow$ Statement 3 is correct

$\therefore$ Statement 1, 2 and 3 are correct.

Solution for Q.no. 85 and 86:

Given: $f(x)$ and $g(x)$ are two functions

$$g(x) = x - \frac{1}{x} \text{ and } fog(x) = x^3 - \frac{1}{x^3}$$

$$f\left(x - \frac{1}{x}\right) = x^3 - \frac{1}{x^3}$$

$$\Rightarrow \qquad f\left(x - \frac{1}{x}\right) = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$$

85. Option (a) is correct.

$$f(x) = x^3 + 3x \qquad \text{...(i)}$$

$$g[f(x) - 3x] = f(x) - 3x - \frac{1}{f(x) - 3x}$$

$$g[f(x) - 3x] = x^3 + 3x - 3x - \frac{1}{x^3 + 3x - 3x}$$

$$= x^3 - \frac{1}{x^3}$$

$$g[f(x) - 3x] = x^3 - \frac{1}{x^3}$$

86. Option (d) is correct.

$$f'(x) = 3x^2 + 3$$

$$f''(x) = 6x$$

Solution for Q.no. 87 and 88:

$$f(x) = |x| + 1 \text{ and } g(x) = [x] - 1,$$

$$h(x) = \frac{f(x)}{g(x)} = \frac{|x|+1}{[x]-1}$$

87. Option (a) is correct.

1. For $x < 0$

$f(x) = -x + 1$

So, $f'(x) = -1,$

So $f(x)$ is differentiable $\forall\ x < 0$

$\Rightarrow$ Statement 1 is correct.

2. At $x = 0.0001$

$g(x)$ is continuous every where except for integer value as $[x]$ is continuous $\forall\ x \in$ R except integers.

So $g(x)$ is continuous at $x = 0.0001$, as 0.0001 is not an integer.

$\Rightarrow$ Statement 2 is correct.

3. $g(x) = [x] - 1$

$[x] \Rightarrow$ Always *ab* integer

$$g'(x) = \frac{d}{dx}[x]-0$$

$g'(x) = 0 - 0 = 0$

$\Rightarrow$ Statement 3 is incorrect.

$\therefore$ 1 and 2 only correct.

88. Option (a) is correct.

$$\lim_{x\to0^-} h(x)+\lim_{x\to0^+} h(x) = \lim_{x\to0^-}\frac{|x|+1}{[x]-1}+\lim_{x\to0^+}\frac{|x|+1}{[x]-1}$$

$$= \frac{0+1}{-1-1}+\frac{0+1}{0-1}$$

$$\begin{Bmatrix}[0-h]=-1\\ [0+h]=0\end{Bmatrix}$$

$$= -\frac{1}{2}-1$$

$$= -\frac{3}{2}$$

89. Option (d) is correct.

$$\phi(a) = \int_a^{a+100\pi}|\sin x|\,dx$$

As period of $|\sin x| = \pi$ and $|\sin x| = \sin x$ in the interval 0 to π.

$$\phi(a) = \int_0^{100\pi}|\sin x|\,dx$$

$$= 100\int_0^{\pi}|\sin x|\,dx$$

$$= 100\int_0^{\pi}\sin x\,dx$$

$$\phi(a) = 100(-\cos x)_0^{\pi}$$

$= 100 \times 2$

$= 200$

90. Option (a) is correct.

$\phi(a) = 200$

$\phi'(a) = 0$

Solution for Q.no. 91 and 92:

Given, $y = 2f(x) + ax - b$ ...(i)

Differentiating given function w.r.t. 'x' we get,

$$\Rightarrow \frac{dy}{dx} = 2f'(x) + a(1) \quad ...(ii)$$

Again differentiating given function, we get,

$$\Rightarrow \frac{d^2y}{dx^2} = 2f''(x) \quad ...(iii)$$

91. Option (c) is correct.

Since, $f(x)$ has a local maximum at $x = 0$. Hence,

$f'(0) = 0$ and $f''(0) < 0$ ...(iv)

Therefore, option (c) is correct.

92. Option (b) is correct.

Since, y has a relative maxima at $x = 0$. Hence,

$$\left(\frac{dy}{dx}\right)_{x=0} = 0$$

$\Rightarrow 2f'(0) + a = 0$ [from (ii)]

$\Rightarrow 2(0) + a = 0$ [from (iv)]

$\Rightarrow a = 0$

also $\left(\dfrac{d^2y}{dx^2}\right)_{x=0} < 0$

$\Rightarrow 2f''(0) < 0$

$\Rightarrow f''(0) < 0$

Clearly y has a relative maxima for $a = 0$ and all values of 'b'.

Solution for Q.no. 93 and 94:

Given, $f(x) = |x - 1|$ ...(i)

$g(x) = [x]$ ...(ii)

and $h(x) = f(x).g(x)$

$\Rightarrow h(x) = |x - 1|.[x]$ ...(iii)

93. Option (a) is correct.

$$\int_{-1}^{0} h(x)dx = \int_{-1}^{0} f(x).g(x).dx$$

$$= \int_{-1}^{0}|x-1|.[x]dx$$

$$= \int_{-1}^{0}(1-x).(-1)dx$$

$$= \int_{-1}^{0}(x-1)dx$$

$$= \left[\frac{x^2}{2}-x\right]_{-1}^{0} = \left[0-\left(\frac{1}{2}+1\right)\right]$$

$$= -\frac{3}{2}$$

94. Option (d) is correct.

$$\int_0^2 h(x)dx = \int_0^1 |x-1|[x]dx + \int_1^2 |x-1|[x]dx$$

$$= \int_0^1 (1-x).(0)dx + \int_1^2 (x-1).(1)dx$$

$$= \int_1^2 (x-1)dx$$

$$= \left[\frac{x^2}{2} - x\right]_1^2$$

$$= \left(\frac{4}{2} - 2\right) - \left(\frac{1}{2} - 1\right)$$

$$= \frac{1}{2}$$

Solution for Q.no. 95 and 96:

Given, $\int \frac{dx}{\sqrt{x+1} - \sqrt{x-1}} = \alpha(x+1)^{\frac{3}{2}} + \beta(x-1)^{\frac{3}{2}} + C$ ...(i)

Let $I = \int \frac{dx}{\sqrt{x+1} - \sqrt{x-1}}$

$$= \int \frac{(\sqrt{x+1} + \sqrt{x-1})dx}{(\sqrt{x+1} - \sqrt{x-1})(\sqrt{x+1} + \sqrt{x-1})}$$

$\Rightarrow$ $I = \frac{1}{2}\int \sqrt{(x+1)}\,dx + \frac{1}{2}\int \sqrt{x-1}\,dx$

$\Rightarrow$ $I = \frac{1}{2}.\frac{(x+1)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + \frac{1}{2}.\frac{(x-1)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C$

$\Rightarrow$ $I = \frac{1}{3}(x+1)^{\frac{3}{2}} + \frac{1}{3}(x-1)^{\frac{3}{2}} + C$

$\Rightarrow \int \frac{dx}{\sqrt{x+1} - \sqrt{x-1}} = \frac{1}{3}(x+1)^{\frac{3}{2}} + \frac{1}{3}(x-1)^{\frac{3}{2}} + C$ ...(ii)

95. Option (a) is correct.

From (i) and (ii) we get

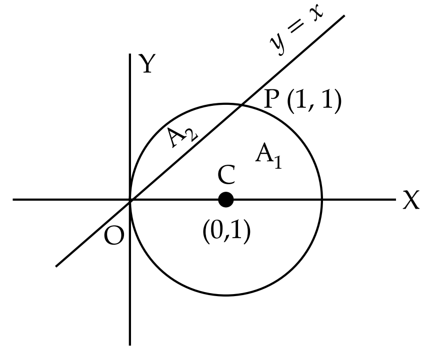
$$\alpha = \frac{1}{3}$$

96. Option (c) is correct.

From (i) and (ii) we get

$$\beta = \frac{1}{3}$$

Solution for Q.no. 97 and 98:



Given, $x^2 + y^2 - 2x = 0$

$\Rightarrow$ $(x-1)^2 + y^2 = 1$

or

$y^2 = 1 - (x-1)^2$

Area of minor segment $(A_2) = \int_0^1 (\sqrt{1-(x-1)^2} - x)dx$

$\Rightarrow A_2 = \left[\frac{(x-1)}{2}\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}(x-1) - \frac{x^2}{2}\right]_0^1$

$\Rightarrow A_2 = \left[\left(0 + \frac{1}{2}(0) - \frac{1}{2}\right) - \left(0 + \frac{1}{2}\sin^{-1}(-1) - 0\right)\right]$

$\Rightarrow A_2 = \frac{\pi - 2}{4}$

97. Option (d) is correct.

Since, area of given circle $= \pi(1)^2 = \pi$

$\Rightarrow$ $A_1 + A_2 = \pi$

$\Rightarrow$ $A_1 = \pi - \frac{\pi - 2}{4}$

$$= \frac{3\pi + 2}{4}$$

98. Option (a) is correct.

$$\frac{2(A_1 + A_2)}{A_1 - 3A_2} = \frac{2(\pi)}{\left(\frac{3\pi + 2}{4}\right) - \frac{3(\pi - 2)}{4}}$$

$$= \pi$$

Solution for Q.no. 99 and 100:

99. Option (d) is correct.

Given, $3f(x) + f\left(\frac{1}{x}\right) = \frac{1}{x} + 1$ ...(i)

Replacing (x) by $\left(\frac{1}{x}\right)$, we get

$3f\left(\frac{1}{x}\right) + f(x) = x + 1$ ...(ii)

Now, on 3 × eq. (i) – eq. (ii), we get

$$8f(x) = \frac{3}{x} + 3 - x - 1$$

$$\Rightarrow \quad f(x) = \frac{3}{8x} - \frac{x}{8} + \frac{1}{4} \quad \text{...(iii)}$$

100. Option (a) is correct.

$$8\int_1^2 f(x)dx = 8\int_1^2 \left(\frac{3}{8x} - \frac{x}{8} + \frac{1}{4}\right)dx$$
$$= \int_1^2 \left(\frac{3}{x} - x + 2\right)dx$$
$$= \left[3\ln x - \frac{x^2}{2} + 2x\right]_1^2$$
$$= \left(3\ln 2 - \frac{4}{2} + 4\right) - \left(3\ln 1 - \frac{1}{2} + 2\right)$$
$$= 3\ln 2 + 2 - \frac{3}{2}$$
$$= \frac{1}{2} + 3\ln 2$$
$$= \frac{1}{2}\ln e + \ln 2^3$$
$$= \ln\sqrt{e} + \ln 8$$
$$= \ln(8\sqrt{e})$$

101. Option (c) is correct.

Number of ways of selecting two black balls from the bag = ${}^5C_2$

Number of ways of selecting two white balls from the bag = ${}^4C_2$

Number of ways of selecting both of the ball of same colour = ${}^5C_2$ + ${}^4C_2$

Now, the required probability is,

$$P[E] = \frac{{}^5C_2 + {}^4C_2}{{}^9C_2} = \frac{10+6}{36}$$
$$P[E] = \frac{16}{36} = \frac{4}{9}$$

102. Option (c) is correct.

Mean = 5 and Variance = 4

$$5^{23}P(X=3) = \lambda 4^{\lambda} \quad \text{...(i)}$$

$\because$ Mean = 5

$\Rightarrow$ $np = 5$

Variance = 4

$\Rightarrow$ $npq = 4$

$\Rightarrow$ $5q = 4$

$$\Rightarrow \quad q = \frac{4}{5}$$

Now, $$p = 1 - q = 1 - \frac{4}{5}$$
$$= \frac{1}{5}$$

$\because$ $np = 5$

$\Rightarrow$ $n = 25$

Now, $5^{23}P(X = 3) = 5^{23}[{}^nC^3\, p^3\, q^{n-3}]$

$$= 5^{23}\left[{}^{25}C_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^{22}\right]$$
$$= 5^{23}\left[\frac{25\times 24\times 23}{6}\times\frac{1}{5^3}\times\frac{4^{22}}{5^{22}}\right]$$
$$= 4 \times 23 \times 4^{22}$$
$$\Rightarrow \quad 5^{23}P(X=3) = 23 \times 4^{23} \quad \text{...(ii)}$$

From equation (i) and (ii):

$$\lambda = 23$$

103. Option (b) is correct.

x	y	xy	x^2
–4	1	–4	16
–1	2	–2	1
2	7	14	4
3	1	3	9

$\Sigma x = -4 - 1 + 2 + 3 = 0$

$\Sigma y = 1 + 2 + 7 + 1 = 11$

$\Sigma xy = -4 - 2 + 14 + 3 = 11$

$\Sigma x^2 = 16 + 1 + 4 + 9 = 30$

$$\bar{x} = \frac{\Sigma x}{4} = \frac{0}{4} = 0$$
$$\bar{y} = \frac{\Sigma y}{4} = \frac{11}{4}$$
$$b_{yx} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{4}}{\Sigma x^2 - \frac{(\Sigma x)^2}{4}}$$
$$= \frac{11-0}{30-0} = \frac{11}{30}$$

Equation of regression line of y on x will be.

$$y - \bar{y} = b_{yx}(x - \bar{x})$$
$$\Rightarrow \quad y - \frac{11}{4} = \frac{11}{30}(x-0)$$
$$\Rightarrow \quad y = \frac{11}{30}x + \frac{11}{4}$$
$$\therefore \quad a = \frac{11}{4} \text{ and } b = \frac{11}{30}$$
$$2a + 15b = 2\times\frac{11}{4} + 15\times\frac{11}{30}$$
$$= 11$$

104. Option (a) is correct.

Slope of line $x + 2y + 1 = 0$

$$\Rightarrow \quad m_1 = -\frac{1}{2}$$

Slope of line $2x + 3y + 4 = 0$

$$\Rightarrow \quad m_2 = -\frac{2}{3}$$

$$\therefore \quad \tan\theta = \left|\frac{\frac{-1}{2}+\frac{2}{3}}{1+\frac{1}{2}\times\frac{2}{3}}\right| = \frac{1}{8}$$

$$\therefore \quad \tan 3\theta = \frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$$

$$= \frac{\frac{3}{8}-\frac{1}{8^3}}{1-\frac{3}{64}} = \frac{192-1}{8^3}\times\frac{8^2}{61}$$

$$= \frac{191}{8\times 61}$$

Now, $\tan 3\theta = 488\times\frac{191}{8\times 61}$

$= 191$

105. Option (a) is correct.

$$\because \quad \frac{2X-3Y}{5X+4Y} = 4$$

$$\Rightarrow \quad 2X-3Y = 20X + 16Y$$

$$\Rightarrow \quad Y = -\frac{18X}{19}$$

$$\text{Var}(X) = npq = np(1-p)$$

$$= 10\times\frac{1}{2}\times\frac{1}{2}$$

$$\Rightarrow \quad \text{Var}(X) = \frac{5}{2}$$

Now, $\text{Var}(Y) = \text{Var}\left(-\frac{18}{19}X\right)$

$$= \left(-\frac{18}{19}\right)^2 \text{Var}(X)$$

$$\Rightarrow \quad \text{Var}(Y) = \left(\frac{18}{19}\right)^2\times\frac{5}{2}$$

$$= \frac{810}{361}$$

106. Option (b) is correct.

$\because$ a, b, c are in H.P.

$$\Rightarrow \quad \frac{2}{b} = \frac{1}{a}+\frac{1}{c}$$

$$\Rightarrow \quad b = \frac{2ac}{a+c}$$

$$b-a = \frac{2ac}{a+c}-a = \frac{2ac-a^2-ac}{a+c}$$

$$= \frac{ac-a^2}{a+c} = \frac{a(c-a)}{a+c}$$

$$\Rightarrow \quad \frac{1}{b-a} = \frac{a+c}{-a(a-c)}$$

Now, $b-c = \frac{2ac}{a+c}-c = \frac{2ac-ac-c^2}{a+c}$

$$= \frac{ac-c^2}{a+c} = \frac{c(a-c)}{a+c}$$

$$\Rightarrow \quad \frac{1}{b-c} = \frac{a+c}{c(a-c)}$$

$$\therefore \quad \frac{1}{b-a}+\frac{1}{b-c} = \frac{a+c}{-a(a-c)}+\frac{a+c}{c(a-c)}$$

$$= \frac{(a+c)}{(a-c)}\left[-\frac{1}{a}+\frac{1}{c}\right]$$

$$= \frac{a+c}{ac} = \frac{1}{a}+\frac{1}{c}$$

$$= \frac{2}{b}$$

$$= \frac{1}{b}+\frac{1}{b} = \frac{1}{b}+\frac{1}{2}\left(\frac{1}{a}+\frac{1}{c}\right)$$

So, only statement 1 and 2 are correct and statement 3 is not correct.

$\therefore$ option (b) matches the given criterion.

107. Option (c) is correct.

The rates of the edible oil in four consecutive years are 150, 200, 250, 300

Now, average price of oil $= \frac{150+200+250+300}{4}$

$$= \frac{900}{4}$$

$$= 225$$

108. Option (b) is correct.

The given word is 'TIRUPATI'.

Let E be the event of being both the T's together.

$\therefore \quad n(E) = 7!$

Now, the required probability is

$$P(E) = \frac{n[E]}{n[S]} = \frac{2!7!}{8!}$$

$$= \frac{2}{8} = \frac{1}{4}$$

109. Option (c) is correct.

$$m = 77^n$$

Let us find some values of m of different values of n.

For $n = 1, m = 77^1 = 77$

For $n = 2, m = 77^2 = 5929$

For $n = 3, m = 77^3$

= Last digit will be 3

For $n = 4, m = 77^4$

= Last digit will be 1.

Now, the same pattern of unit digit will be obtained for other values of n.

Thus, we observe that unit digit of m is 1 after each 4 values of n.

Hence, the required probability $= \frac{1}{4}$

110. Option (d) is correct.

$$n(S) = {}^{15}C_3$$

$\because$
$$2 \times 3 = 6$$
$$2 \times 4 = 8$$
$$2 \times 5 = 10$$
$$2 \times 6 = 12$$
$$2 \times 7 = 14$$
$$3 \times 4 = 12$$

So, we have 6 cases.

$\Rightarrow \quad n(E) = 6$

So, the required probability

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{{}^{15}C_3}$$

$$= \frac{6}{\frac{15 \times 14 \times 13}{6}}$$

$$= \frac{36}{15 \times 14 \times 13}$$

$$\Rightarrow \quad P(E) = \frac{6}{455}$$

Solution for Q.no. 111 and 112:

Given that, $P(A \cup B) \geq 0.75$ and
$0.125 \leq P(A \cap B) \leq 0.375$

111. Option (d) is correct.

$P(A \cup B) \geq 0.75$

$\Rightarrow P(A) + P(B) - P(A \cap B) \geq 0.75$

$\Rightarrow P(A) + P(B) \geq 0.75 + P(A \cap B)$

$\Rightarrow P(A) + P(B) \geq 0.75 + 0.125$

$\{\because P(A \cap B) \geq 0.125\}$

$\Rightarrow P(A) + P(B) \geq 0.875$

Minimum value of

$P(A) + P(B) = 0.875$

112. Option (b) is correct.

Since, $P(A \cap B) \leq 0.375$

$\Rightarrow P(A) + P(B) - P(A \cup B) \leq 0.375$

$\Rightarrow P(A) + P(B) \leq 0.375 + P(A \cup B)$

$\Rightarrow P(A) + P(B) \leq 0.375 + 0.75$

$\{\because P(A \cup B) \geq 0.75\}$

$\Rightarrow P(A) + P(B) \leq 1.125$

$\Rightarrow$ Maximum value of $P(A) + P(B)$ is 1.125.

Solution for Q.no. 113 and 114:

$P(A) = 0.6, \quad P(B) = 0.4, \quad P(C) = 0.5,$
$P(A \cup B) = 0.8, \; P(A \cap C) = 0.3, \; P(A \cap B \cap C) = 0.2$
and $P(A \cup B \cup C) \geq 0.85$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= 0.6 + 0.4 - 0.8 = 0.2$$

Now, $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cup B)$
$- P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$$= 1.5 - 0.2 - 0.3 - P(B \cap C) + 0.2$$

$\Rightarrow \quad P(A \cup B \cup C) = 1.2 - P(B \cap C)$

Since, $\quad 0.85 \leq P(A \cup B \cup C) \leq 1$

$\Rightarrow \quad 0.85 \leq 1.2 - P(B \cap C) \leq 1$

$\Rightarrow \quad -0.35 \leq -P(B \cap C) \leq -0.2$

$\Rightarrow \quad 0.2 \leq P(B \cap C) \leq 0.35$

113. Option (b) is correct.

Minimum value of $P(B \cap C) = 0.2$

114. Option (c) is correct.

Maximum value of $P(B \cap C) = 0.35$

Solution for Q.no. 115 and 116:

p = Probability of getting at least one tail.

$$= 1 - \left(\frac{1}{2}\right)^n$$

q = Probability of getting at least two tails.

$$= 1 - \left(\frac{1}{2}\right)^n - \frac{n}{2^n}$$

115. Option (b) is correct.

$$p - q = \frac{5}{32}$$

$$\Rightarrow \quad 1 - \left(\frac{1}{2}\right)^n - \left[1 - \left(\frac{1}{2}\right)^n - \frac{n}{2^n}\right] = \frac{5}{32}$$

$$\Rightarrow \quad \frac{n}{2^n} = \frac{5}{32}$$

or
$$\frac{n}{2^n} = \frac{5}{2^5}$$

Comparing both sides, we get, $n = 5$

116. Option (a) is correct.

$$p + q = 2 - 2\left(\frac{1}{2}\right)^n - \frac{n}{2^n}$$

$$= 2 - 2 \times \frac{1}{2^5} - \frac{5}{2^5}$$

$$= \frac{57}{32}$$

Solution for Q.no. 117 and 118:

117. Option (b) is correct.

$$\Sigma x_i f_i = 1.1 + \frac{2}{2} + \frac{3}{2^2} + \ldots + \frac{n}{2^{(n-1)}} \quad \ldots(i)$$

and $$\frac{1}{2}\Sigma x_i f_i = \frac{1}{2} + \frac{2}{2^2} + \ldots + \frac{n-1}{2^{n-1}} + \frac{n}{2^n} \quad \ldots(ii)$$

Eq (i) – (ii) we get

$$\Rightarrow \left(1 - \frac{1}{2}\right)\Sigma x_i f_i$$

$$= \left[1.1 + \frac{2-1}{2} + \frac{3-2}{2^2} + \frac{4-3}{2^3} + \ldots + \frac{1}{2^{n-1}}\right] - \frac{n}{2^n}$$

$$\Rightarrow \frac{1}{2}\Sigma x_i f_i = \left[1 + \frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^4} + \ldots + \frac{1}{2^{n-1}}\right] - \frac{n}{2^n}$$

$$\Rightarrow \frac{1}{2}\Sigma x_i f_i = \frac{1\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} - \frac{n}{2^n}$$

$$= \frac{2(2^n - 1)}{2^n} - \frac{n}{2^n}$$

$$\Rightarrow \Sigma x_i f_i = \frac{2^{n+1} - n - 2}{2^{n-1}}$$

118. Option (c) is correct.

$$\Sigma f_i = 1 + \frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^{(n-1)}}$$

$$\Sigma f_i = \frac{\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} = \frac{2^n - 1}{2^{n-1}}$$

Now, $$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{2^{n+1} - n - 2}{2^{n-1}} \cdot \frac{2^{n-1}}{2^n - 1}$$

$$= \frac{2^{n+1} - n - 2}{2^n - 1}$$

Solution for Q.no. 119 and 120:

Ascending data is, 15, 18, 19, 21, 24, 32, 35, 41, 47, 50

Largest 5 observations are, 32, 35, 41, 47, 50

Mean $$\mu = \frac{32 + 35 + 41 + 47 + 50}{5}$$

$$= \frac{205}{5}$$

$$\Rightarrow \mu = 41$$

119. Option (c) is correct.

$$\text{Mean deviation} = \frac{\Sigma |x_i - \mu|}{n}$$

$$= \frac{|32 - 41| + |35 - 41| + |41 - 41| + |47 - 41| + |50 - 41|}{5}$$

$$= \frac{9 + 6 + 0 + 6 + 9}{5}$$

$$= 6$$

120. Option (d) is correct.

$$\sigma^2 = \frac{\Sigma(x_i - \mu)^2}{5}$$

$$= \frac{9^2 + 6^2 + 0^2 + 6^2 + 9^2}{5}$$

$$= \frac{234}{5}$$

$$= 46.8$$